On the Growth of Layers of Nonprecipitating Cumulus Convection

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Introduction - Motivation

Interested in the time evolution of the depth of layers of shallow cumulus convection i.e. $\frac{dh}{dt}$

- Onset of precipitation closely tied to the depth of the cloud layer.
- Impact on surface fluxes affects larger-scale circulations
- Medium-range weather forecasts
- Diurnal cycle of convection over land
- Much of the variance in climate sensitivity attributed to differences in response of layers of shallow convection

Introduction - Idealized view

Buoyancy point of view:

Turbulence from surface buoyancy flux B_0 is converted to potential energy at top with a rate measured by the entrainment buoyancy flux B_h.

In idealized situations these buoyancy fluxes form a fixed fraction i.e.

$$\frac{B_h}{B_0} = k = \text{ entrainment flux ratio, often } = -0.2 \tag{1}$$

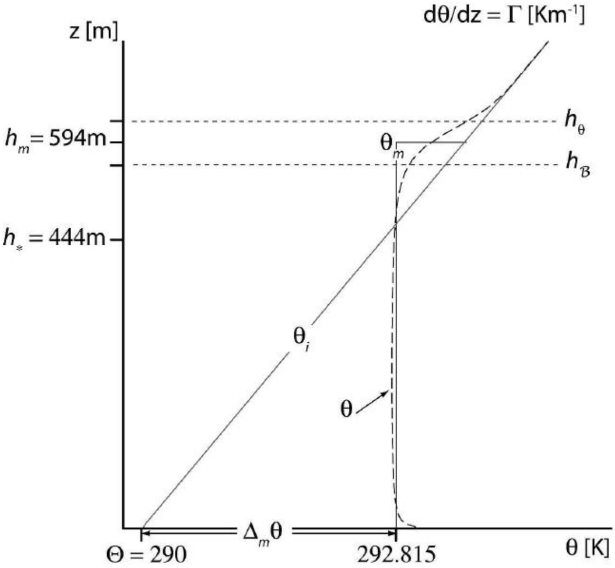
Setup: Background - Dry convective boundary layer

Assumptions: uniformly stratified fluid, constant B_0, mixed layer growth equals convective layer growth, evolution independent of Prandtl and Reynolds number and $(tN)^{1/3}$

Parameters: Stratification N^2, B_0, time t.

Time evolution:

$$h_m = \underbrace{\sqrt{1-2k}}_{:=\alpha} \underbrace{\left(\frac{2B_0 t}{N^2}\right)^{1/2}}_{:=h_*} \tag{2}$$



h_m height of the mixed layer,
h_* encroachment depth,
h_B height of the minimum buoyancy flux,
h_θ height of the maximum buoyancy gradient

New assumptions: Surface temperature varies to keep B_0 constant, initial humidity profile:

$$q_i(z) = q_{0,+} \exp\left[-z/\lambda\right] \tag{3}$$

New parameters: λ, q_0, moisture scale height:

$$\lambda_0 = \frac{R_v c_p \Theta^2}{Lg} \tag{4}$$

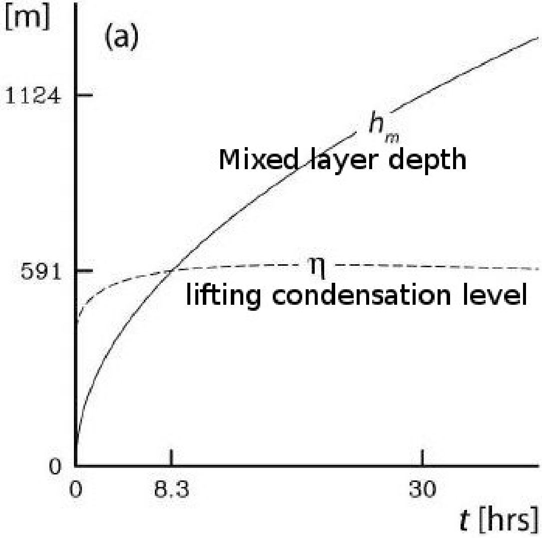
New time evolution (conservation laws):

$$h_m \theta_m = \int_0^{h_m} \theta_i dz + \int_0^t Q_0 dt \tag{6}$$

$$h_m q_m = \int_0^{h_m} q_i dz + \int_0^t R_0 dt \tag{7}$$

$$Q_0 = V\left(\theta_0 - \theta_{0,+}\right) \text{ and } R_0 = V\left(q_0 - q_{0,+}\right) \tag{5}$$

Setup: Prototype problem - Development of TBL

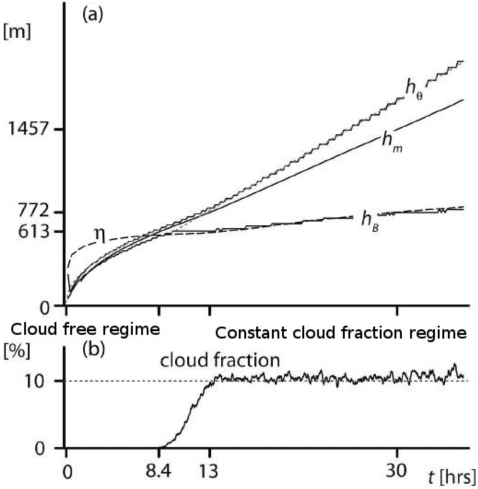


Prognostic variables: Velocity vector, total water mixing ratio q, liquid-water potential temperature:

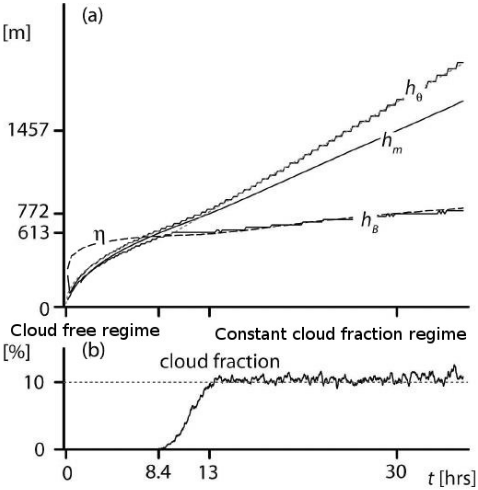
$$\theta_l = \theta \exp\left[-q_l L / \left(c_p T\right)\right]$$

Diagnosed variables: Liquid water q_l

Large-eddy simulations: Time evolution

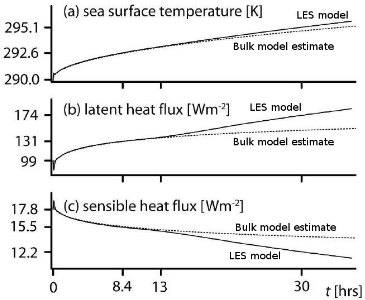


Large-eddy simulations: Time evolution



Q: Why does the cloud fraction equilibrate? Why at 10 % or on which parameters does this depend? Is this realistic?

Large-eddy simulations: Bowen ratio



The Bowen ratio: sensible heat flux / latent heat flux

$$\beta = \frac{c_p Q_0}{L R_0} \text{ with } Q = \overline{w'\theta'} \text{ and } R = \overline{w'q'}$$

The surface buoyancy flux:

$$B_0 = \frac{\theta_0}{g}\left(1 + a_2 \frac{c_p T}{\beta L}\right) V\left(\theta_0 - \theta_{0,+}\right) \text{ with } a_2 = \frac{R_v}{R_d} - 1 \qquad (8)$$

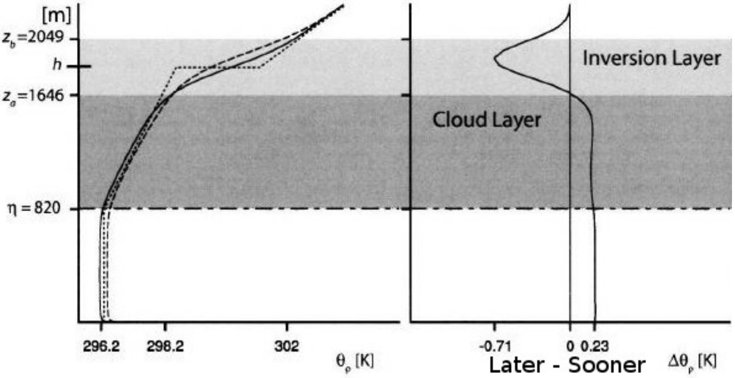
Large-eddy simulations: Implications

With only dry convection compared to shallow moist convection would be:

- Shallower trade winds
- Less surface heat fluxes . . .
- . . . and thus less cooling of the surface ocean
- Less moistening of the atmosphere

Energetics of growing cloud layer: Theoretical framework

$$\theta_\rho = \theta \left[1 + \left(\frac{R_v}{R_d} - 1 \right) q - \frac{R_v}{R_d} q_l \right] \quad (9)$$



Energetics of growing cloud layer: Ballistic growth

Linearly relate fluctuations of the different variables:

$$\theta'_\rho = a_1\theta'_l + a_2\Theta q' + a_3\Theta q'_l \tag{10}$$

will lead after some steps to:

$$\widetilde{Q}_\rho(z) \approx \widetilde{Q}_{\rho,0}\left[1-(1-k)\frac{z}{\eta}\right] \text{ for } z \le z_a \tag{20}$$

which corresponds to a linear extrapolation from the surface value and the top of the subcloud layer of the part of the buoyancy flux that is not associated with perturbations in q_l.

Energetics of growing cloud layer: Assumptions and Implications

New or renewed assumptions in between the steps on last slide:

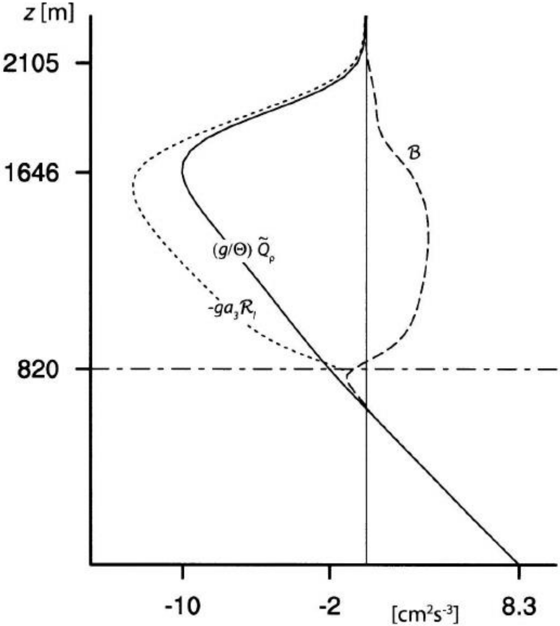
- Neglect fluctuations among thermodynamic perturbations (a_1 and a_3 constants).
- No radiation and precipitation.
- Buoyancy in saturated flow only conserved if q_l is stationary.
- For unsaturated flow $\widetilde{Q}_\rho = Q_\rho = (\Theta/g)\, B$

Implications from these new equations:

Clouds only equilibrate the density of the cloud layer to that of the subcloud layer.

Cloudy layers reduce energetic constraint so that the $\widetilde{Q}_\rho$ can scale with height.

Energetics of growing cloud layer: Flux relationship



Conservation law under assumption z_a and z_b evolve at same rate:

$$\frac{d}{dt}\int_{z_z(t)}^{z_b(t)} \theta_\rho dz - \frac{dh}{dt}\left[\theta_\rho\left(z_b\right) - \theta_\rho\left(z_a\right)\right] = \widetilde{Q}_\rho\left(z_a\right) - \widetilde{Q}_\rho\left(z_b\right) \quad (21)$$

then assume further that $\widetilde{Q}_\rho\left(z_b\right)$ is negligible and that the left hand side is proportional to $\Delta\theta_\rho \frac{dh}{dt}$ and also that the lapse rate in the cloud layer is constant Γ_c, will result in:

Energetics of growing cloud layer: Detailed budget prediction

$$\frac{dh}{dt} = \kappa \left[\frac{\widetilde{Q}_{\rho,0} \left[1 - (1-k)\frac{h}{\eta} \right]}{\theta_{\rho,0} + \Gamma h - \left[\hat{\theta}_{\rho} + \Gamma_c (h - \eta) \right]} \right] \tag{26}$$

with κ being a non-dimensional proportionality constant and $\hat{\theta}_{\rho}$ is the mean up to the cloud base height.

Discussion

Assumption to explain why θ_ρ in cloud layer tracks its subcloud layer values, cloud and subcloud are moistening at the same rate allows partition of moisture and heat flux divergences.

Linear growth only for constant surface buoyancy flux and uniform outer layer stratification.

Processes that reduce the growth: Large-scale subsidence, precipitation and unfavorable gradients in θ_e.

Especially **precipitation**, which can be efficient at depleting the liquid-water flux and also increases Γ_c.

Summary - Process description

Before clouds form: Identical to dry convection.
After clouds form: More rapid deepening, enhanced downward mixing, more moisture fluxes.

Rapid deepening from injection of liquid water into an conditionally unstable capping layer, measured by $\widetilde{Q}_\rho$ and thus constrained by subcloud layer energetics and the tendency of θ_ρ values in the cloud layer to change proportional to its values in the subcloud layer.

Summary - Idealizations

- No precipitation
- No radiative cooling
- No vertical shear in horizontal wind
- Bowen ratio have to decrease strong enough to keep B_0 constant