

**651-2124-00:**

**Atmospheric General Circulation Dynamics** (due March 21):

**Momentum and angular momentum balances.**

1. Reynolds decomposition

- (a) Show that for a zonal average  $\overline{(\cdot)}$ , any product  $AB$  for scalar fields  $A$  and  $B$  can be decomposed as

$$\overline{AB} = \overline{A}\overline{B} + \overline{A'B'}, \quad (1)$$

where primes denote deviations from the zonal mean (i.e., eddies),  $(\cdot)' = (\cdot) - \overline{(\cdot)}$ .

- (b) Under which circumstances does the decomposition (1) hold when  $\overline{(\cdot)}$  denotes a time average?

2. Momentum balance

- (a) Write down the zonal and meridional momentum balance equations and average them zonally (along latitude circles), decomposing quadratic terms into mean and eddy components.
- (b) What are characteristic scales of the individual terms for large-scale flows? Define a horizontal length scale  $L = O(1000 \text{ km})$ , a fluctuating velocity scale  $U = O(10 \text{ m s}^{-1})$ , etc. What is the ratio of the advection of momentum by the mean flow and the Coriolis force? What is this ratio called?
- (c) Why are zonal-mean zonal velocities on rapidly rotating planets so much larger than zonal-mean meridional velocities?
- (d) Under what conditions can zonal-mean meridional flow arise?

3. Angular momentum balance

- (a) Explain (simply and in words) why the angular momentum component about the spin axis can be written as  $M = \Omega r_{\perp}^2 + ur_{\perp} = M_{\Omega} + M_u$ , where  $r_{\perp} = r \cos \phi$  (radius  $r$  and latitude  $\phi$ ).
- (b) Write down the balance equation for  $M$  and average it zonally. (You do not need to derive it.) Show how to obtain the zonal momentum equation from the angular momentum equation.

- (c) Show that the advection of the planetary component of the angular momentum,  $\mathbf{u} \cdot \nabla M_\Omega$ , represents a Coriolis torque.
- (d) From an angular momentum perspective, what is required for a zonal-mean meridional flow to arise? Why does the planetary rotation inhibit zonal-mean meridional flow but not zonal flow?
- (e) By integrating the angular momentum balance along surfaces of constant  $M_\Omega$ , show that in a statistically steady state there can be no net torque on an  $M_\Omega$  surface, exerted either by turbulent angular momentum transfer (Reynolds stress) or drag forces.
- (f) Explain briefly how this integral constraint from the angular momentum balance constrains zonal winds.

Each of the above problems can be answered relatively briefly: 2(a)–(d) and 3(a)–(d) should be no more than a few lines each; 3(e) and (f) might take one or two paragraphs each. If you find it easier and more comfortable, you can answer all of the questions under 3 in the thin-atmosphere approximation typically used for Earth (approximating  $r$  by the constant mean planetary radius  $a$ ), but it is not necessary to do so. (Answering the above questions is not any more difficult without that approximation.)