651-2124-00:

Atmospheric General Circulation Dynamics (due March 21):

Momentum and angular momentum balances.

1. Reynolds decomposition

   (a) Show that for a zonal average $\overline{(\cdot)}$, any product AB for scalar fields A and B can be decomposed as

   $$\overline{AB} = \bar{A}\bar{B} + \overline{A'B'}, \tag{1}$$

   where primes denote deviations from the zonal mean (i.e., eddies), $(\cdot)' = (\cdot) - \overline{(\cdot)}$.

   (b) Under which circumstances does the decomposition (1) hold when $\overline{(\cdot)}$ denotes a time average?

2. Momentum balance

   (a) Write down the zonal and meridional momentum balance equations and average them zonally (along latitude circles), decomposing quadratic terms into mean and eddy components.

   (b) What are characteristic scales of the individual terms for large-scale flows? Define a horizontal length scale $L = O(1000\,\mathrm{km})$, a fluctuating velocity scale $U = O(10\,\mathrm{m\,s^{-1}})$, etc. What is the ratio of the advection of momentum by the mean flow and the Coriolis force? What is this ratio called?

   (c) Why are zonal-mean zonal velocities on rapidly rotating planets so much larger than zonal-mean meridional velocities?

   (d) Under what conditions can zonal-mean meridional flow arise?

3. Angular momentum balance

   (a) Explain (simply and in words) why the angular momentum component about the spin axis can be written as $M = \Omega r_\perp^2 + u r_\perp = M_\Omega + M_u$, where $r_\perp = r\cos\phi$ (radius r and latitude ϕ).

   (b) Write down the balance equation for M and average it zonally. (You do not need to derive it.) Show how to obtain the zonal momentum equation from the angular momentum equation.

(c) Show that the advection of the planetary component of the angular momentum, $\mathbf{u} \cdot \boldsymbol{\nabla} M_\Omega$, represents a Coriolis torque.

(d) From an angular momentum perspective, what is required for a zonal-mean meridional flow to arise? Why does the planetary rotation inhibit zonal-mean meridional flow but not zonal flow?

(e) By integrating the angular momentum balance along surfaces of constant M_Ω, show that in a statistically steady state there can be no net torque on an M_Ω surface, exerted either by turbulent angular momentum transfer (Reynolds stress) or drag forces.

(f) Explain briefly how this integral constraint from the angular momentum balance constrains zonal winds.

Each of the above problems can be answered relatively briefly: 2(a)–(d) and 3(a)–(d) should be no more than a few lines each; 3(e) and (f) might take one or two paragraphs each. If you find it easier and more comfortable, you can answer all of the questions under 3 in the thin-atmosphere approximation typically used for Earth (approximating r by the constant mean planetary radius a), but it is not necessary to do so. (Answering the above questions is not any more difficult without that approximation.)