ESE 134: Cloud and Boundary Layer Dynamics (HW 5, due May 29):

Stratocumulus mixed-layer model. A stratocumulus mixed-layer model can be written in terms of three unknowns: the height of the mixed-layer (h), and two slab (mixed-layer) average moist conserved variables: specific total water (q) and liquid water static energy ($s = c_pT + gz - Lq_l$):

$$h\frac{ds}{dt} = V(s_0 - s) + E(s^+ - s) - \frac{\Delta R}{\rho}$$

$$h\frac{dq}{dt} = V(q_0 - q) + E(q^+ - q)$$

$$\frac{dh}{dt} = E - Dh$$

Here, s_0 and q_0 are the surface values, s^+ and q^+ are the values in the free troposphere, V is a turbulent wind velocity close to the surface ($V = C_dU$, where C_d is the drag coefficient and U is the wind speed), D is the divergence, ΔR is the radiative cooling, and ρ is the density of air.

To close the model, an equation for the entrainment rate E is necessary. Since the turbulence in the stratocumulus mixed-layer is mainly driven by cloud-top radiative cooling, we will use a simple and physically plausible formulation, which suggests that the entrainment forced by turbulent mixing is proportional to the rate of the driving of the flow (radiative cooling ΔR) and inversely proportional to the stability of the interface at cloud top, with proportionality constant $\alpha \sim O(1)$, i.e.,

$$E = \alpha\frac{\Delta R/\rho}{s^+ - s}$$

1. Find the steady state analytic solutions of this system of equations for the particular case of $\alpha = 1$, and for a generic α.

2. What are the specific solutions for typical parameter values: $V = 0.01\ \mathrm{m\ s^{-1}}$, $D = 5 \times 10^{-6}\ \mathrm{s^{-1}}$, $\Delta R = 50\ \mathrm{W\ m^{-2}}$, $\rho = 1\ \mathrm{kg\ m^{-3}}$, $s^+ - s = 10\ \mathrm{kJ\ kg^{-1}}$, $q^+ = 5\ \mathrm{g\ kg^{-1}}$, $\mathrm{SST} = 293\ \mathrm{K}$, and $q_0 = q_s(\mathrm{SST}) = 14\ \mathrm{g\ kg^{-1}}$?

3. Investigate the sensitivity of the solutions to changes in the parameters D, V, ΔR and s^+. *Suggestion: compute derivatives of the 3 unknowns (s, q and h) with respect to parameters D, V, ΔR and s^+.*

4. Do these sensitivities make physical sense? Please explain qualitatively.