GEOPHYSICAL RESEARCH LETTERS, VOL. 35, L23803, doi:10.1029/2008GL035847, 2008

Extent of Hadley circulations in dry atmospheres

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Received 28 August 2008; revised 21 October 2008; accepted 27 October 2008; published 2 December 2008.

[1] The subtropical terminus of the Hadley circulation is interpreted as the latitude poleward of which vertical wave activity fluxes (meridional eddy entropy fluxes) become sufficiently deep to reach the upper troposphere. This leads to a sign change of the upper-tropospheric divergence of meridional wave activity fluxes (convergence of meridional eddy angular momentum fluxes) and marks the transition from the tropical Hadley cell to the extratropical Ferrel cell. A quantitative formulation for determining the depth of vertical wave activity fluxes and thus the terminus of the Hadley circulation is proposed based on the supercriticality, a measure of the slope of isentropes. The supercriticality assumes an approximately constant value at the terminus of the Hadley circulation in a series of simulations with an idealized dry general circulation model. However, it is unclear how to generalize this supercriticality-based formulation to moist atmospheres. **Citation:** Korty, R. L., and T. Schneider (2008), Extent of Hadley circulations in dry atmospheres, *Geophys. Res. Lett.*, *35*, L23803, doi:10.1029/2008GL035847.

1. Introduction

[2] Several studies have reported an increase in the meridional extent of the Hadley circulation (HC) in both recent observational data [e.g., *Hu and Fu*, 2007; *Seidel and Randel*, 2007; *Seidel et al.*, 2008] and simulations of 21st-century climate [e.g., *Lu et al.*, 2007; *Seager et al.*, 2007]. Such shifts can dramatically alter climate on regional scales, underscoring the need to understand the dynamical mechanisms responsible. While it has long been suspected that the HC extent may be related to baroclinic eddy activity, an expression for its meridional extent built on a dynamically consistent foundation has not been developed.

[3] Axisymmetric HCs for Earth-like planets are baroclinically unstable, which has led to formulations for HC extent that combine baroclinic instability measures with expressions for the flow within HCs. For example, combining the zonal wind obtained by assuming angular momentum conservation of the meridional flow in the upper troposphere [*Held and Hou*, 1980] with the critical shear for baroclinic instability in the quasigeostrophic two-layer model [*Phillips*, 1954] yields an expression for the HC terminus as the latitude at which the HC would become baroclinically unstable in the two-layer model [*Held*, 2000]. However, the critical shear is an artifact of the vertical truncation of the two-layer model; in a continuously stratified atmosphere, there is none. While some other baroclinic instability measure could be used in its place (e.g., the growth rate from the Eady or Charney model), an additional problem is that upper-tropospheric flows in HCs across a range of climates including Earth's deviate substantially from angular momentum conservation [*Walker and Schneider*, 2006; *Schneider*, 2006], rendering the physical basis for this formulation dubious. Existing theories do not capture the quantitative dependence of HC extent on the static stability and other mean-flow quantities [*Walker and Schneider*, 2006; *Schneider*, 2006].

[4] Here we propose a new formulation built on a foundation that neither requires angular momentum conservation of the tropical upper-tropospheric flow nor uses an expression for baroclinic instability that is predicated on the architecture of a particular model. We take as the defining characteristic of the HC's subtropical terminus that there the divergence of meridional eddy angular momentum fluxes in the upper troposphere changes sign. Upper-tropospheric divergence of eddy angular momentum fluxes near the terminus and poleward thereof is balanced primarily by the Coriolis torque on (Eulerian) mean meridional mass fluxes because the Rossby number there is small [e.g., *Walker and Schneider*, 2006; *Bordoni and Schneider*, 2008]. Where there is eddy angular momentum flux divergence (in the Hadley cells), there is poleward mass flux; where there is convergence (in the Ferrel cells), there is equatorward mass flux. Convergence of meridional eddy angular momentum fluxes is tantamount to divergence of meridional wave activity fluxes, and the wave activity diverging poleward of the HC terminus is brought into the upper troposphere by vertical fluxes, which are related to meridional eddy entropy fluxes [e.g., *Edmon et al.*, 1980]. Therefore, the HC terminus can be interpreted as the latitude poleward of which eddy entropy fluxes are sufficiently deep to reach the upper troposphere, leading to wave activity flux divergence and thus to eddy angular momentum flux convergence and a meridional mass flux that opposes that of the HCs.

[5] The preceding discussion suggests that the HC terminus may be characterized by a critical $O(1)$ value of the supercriticality

$$S_c = -\frac{f}{\beta}\frac{\partial_y \overline{\theta}_s}{\Delta_v} \sim \frac{\overline{p}_s - \overline{p}_e}{\overline{p}_s - \overline{p}_t}, \tag{1}$$

which is a nondimensional measure of the pressure range over which eddy entropy fluxes in dry atmospheres extend [*Schneider and Walker*, 2006; *Schneider*, 2007]. The supercriticality generalizes a similar measure of the depth of eddy entropy fluxes proposed in the context of quasigeostrophic theory by *Held* [1978]. The fields entering (1) are temporal and zonal means: the surface or near-surface potential temperature $\overline{\theta}_s$; the pressures at the surface

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0094-8276/08/2008GL035847$05.00

($\overline{p}_s$), at the tropopause ($\overline{p}_t$), and at the level up to which eddy entropy fluxes extend ($\overline{p}_e$); and the bulk stability

$$\Delta_v = -2\,\overline{\partial_p \theta}^s (\overline{p}_s - \overline{p}_t), \quad (2)$$

which depends on the static stability $-\overline{\partial_p \theta}^s$ near the surface. Although the derivation of S_c as a measure of the depth of eddy entropy fluxes relative to the tropopause height is based on diffusive eddy flux closures, and therefore may be expected only to hold on length scales large compared with those of eddies [*Schneider and Walker*, 2006], here we evaluate S_c locally in latitude, using local values of f, β, and the mean-flow quantities. Formulated this way, the latitude poleward of which S_c first exceeds a critical $O(1)$ value may be that at which eddy entropy fluxes first reach the upper troposphere and, therefore, where the eddy angular momentum flux divergence in the upper troposphere changes sign.

[6] We use simulations with an idealized dry GCM that span a wide range of climates to investigate how the HC extent depends on mean-flow quantities and how the specific formulation proposed here performs. The dry GCM allows us to test theories based on scaling laws for baroclinic eddies in dry atmospheres, postponing questions of how they may be generalized to moist atmospheres. We show that S_c indeed assumes an approximately constant value at the HC's terminus in all but a few simulations.

2. Methods

[7] We use the simulations described in *Walker and Schneider* [2006] and *Schneider and Walker* [2006]. The idealized GCM is a hydrostatic, primitive-equation model with Newtonian relaxation of temperatures toward a radiative-equilibrium state of a semigray atmosphere. The lower boundary is uniform, thermally insulating, and has constant roughness length. Phase changes of water are not explicitly taken into account, but a quasi-equilibrium convection scheme maintains a minimum static stability by relaxing temperatures toward a specified lapse rate $\gamma\Gamma_d$ whenever an air parcel lifted from the lowest model level has positive convective available potential energy relative to the specified profile ($\Gamma_d = g/c_p$ is the dry-adiabatic lapse rate, and γ is a rescaling parameter). For $\gamma = 1$, the convection scheme represents dry convection; for $\gamma < 1$, the convection scheme mimics the stabilizing effect of latent heat release in moist convection, with the implicit latent heat release increasing as γ decreases. The scaling parameter γ effectively controls the thermal stratification of the tropical free troposphere. The radiative-equilibrium state toward which temperatures are relaxed is temporally constant, zonally and hemispherically symmetric, and has a $\Delta_h \cos^2\phi$-dependence of surface temperature on latitude ϕ, with a specified pole-to-equator temperature contrast Δ_h. The simulations have spectral resolutions between T42 and T127 in the horizontal and 30 σ levels in the vertical. For additional model details and simulation descriptions, see *Schneider and Walker* [2006] and *Walker and Schneider* [2006].

[8] We analyze series of simulations in which the convective rescaling parameter is set to values $\gamma = 0.6,\ldots,1.0$. For each value of γ, the pole-to-equator surface temperature contrast in radiative equilibrium is varied between $\Delta_h = 15$ to 30 K at the lower end (depending on γ) and $\Delta_h = 360$ K at the upper end; the resulting pole-to-equator surface temperature contrasts in dynamic equilibrium range between 12 and 104 K. We also analyze series of simulations in which the planetary radius and rotation rate are set to values twice and four times those of Earth, while $\gamma = 0.7$, and Δ_h is varied between 30 K and 360 K. We determine the HC extent using the same criterion as used by *Walker and Schneider* [2006]: the subtropical terminus is taken to be the first latitude poleward of the maximum absolute value of the HC streamfunction at which, at the σ level of its extremum above $\sigma = 0.7$, it is 10% of its extremal value.

3. Hadley Circulation Extent and Supercriticality

[9] The HC extent for these simulations was presented by *Walker and Schneider* [2006], and the latitudes of its terminus are replotted in Figure 1a. The terminus varies with convective rescaling parameter γ and with planetary radius and rotation rate, though for sufficiently large temperature contrast Δ_h, it varies only weakly with Δ_h. Simulations with convective lapse rates that are nearly moist adiabatic for present-day Earth ($\gamma \lesssim 0.7$) have a statically more stable tropical thermal stratification and wider HCs than those with dry-adiabatic convective lapse rates ($\gamma = 1$).

[10] In each of the simulations, S_c increases rapidly from about zero in the deep tropics to $O(1)$ values in the extratropics. Because S_c is evaluated locally (using the conventions of *Schneider and Walker* [2006] for the evaluation of mean-flow quantities), it can exceed 1 in extratropical latitudes, although large-scale extratropical averages do not exceed 1 substantially [*Schneider and Walker*, 2006]. Figure 1b shows S_c at the HC terminus. For the majority of the simulations, S_c falls in the range around 0.6 to 0.7 at the HC terminus, independent of γ, Δ_h, rotation rate, and planetary radius. This suggests that the reasoning laid out in the introduction—that the HC extends to the latitude at which eddy entropy fluxes reach the upper troposphere and that this latitude can be determined based on S_c—is generally adequate.

[11] For simulations with small Δ_h, the value of S_c at the HC terminus varies considerably and is often small. In these simulations, eddies are relatively weak and shallow, and at least in some of them, the HC terminates at a latitude equatorward of any substantial baroclinic eddy activity [*Walker and Schneider*, 2006]. Other exceptions to the general behavior discussed above include the simulations with $\gamma = 1.0$. Here bulk stabilities tend to zero, and S_c becomes singular.

[12] The results in Figure 1b suggest that the HC terminus is indeed characterized by a critical $O(1)$ value $S_c = c$. Because S_c increases monotonically from the equator into the subtropics, the HC terminus can be determined approximately from the mean thermal structure of the atmosphere as the latitude at which S_c first exceeds this critical value c. Figure 1c shows the separation between the actual HC terminus and the latitude at which S_c first exceeds $c = 0.63$ (the mean value of S_c at the HC terminus for all simulations with $\Delta_h \geq 60$ K and $\gamma < 1.0$). For all cases except those with very small Δ_h or when $\gamma = 1.0$, the actual location of the HC terminus is well characterized by this critical value for S_c.

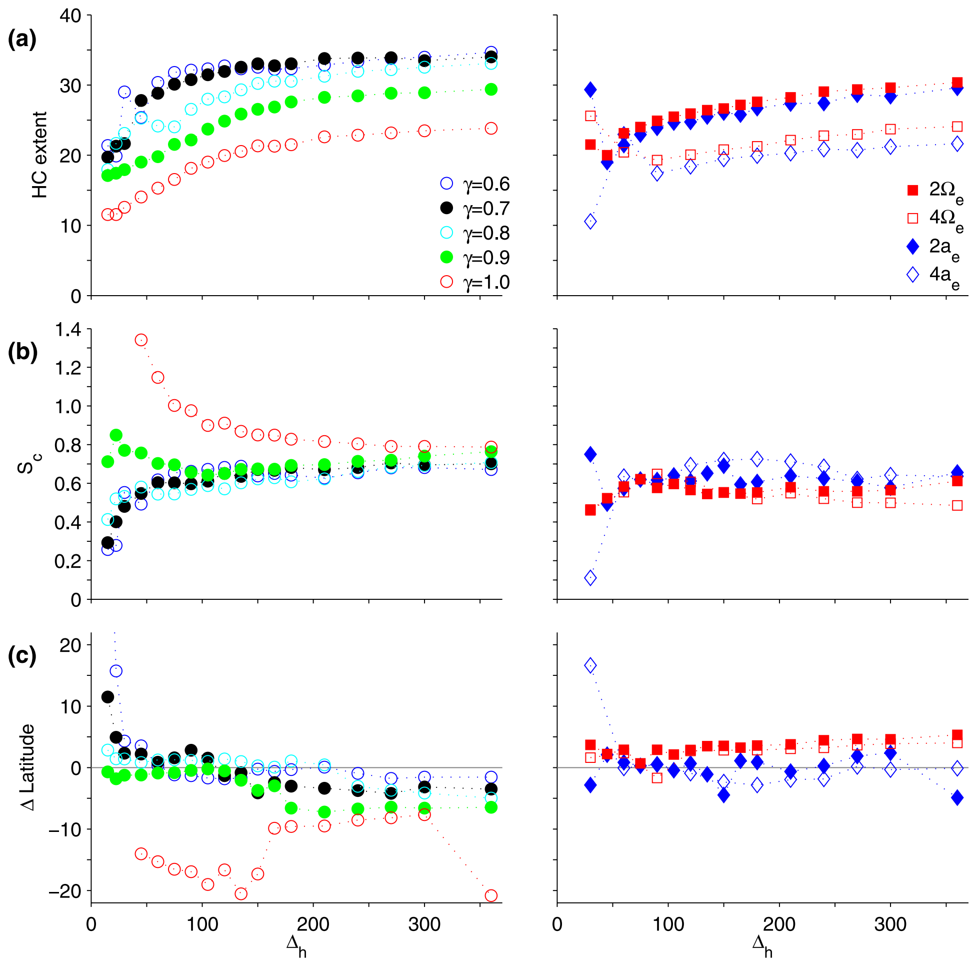

Figure 1. (a) Terminus of Hadley circulation as a function of Δ_h. (b) Locally assessed supercriticality at HC terminus. (c) Difference between HC terminus expected according to $S_c = 0.63$ and actual terminus. (left) Simulations with different convective rescaling parameter γ and Earth's radius and rotation rate. (right) Simulations with $\gamma = 0.7$ and 2 and 4 times Earth's radius a_e and rotation rate Ω_e.

[13] Both $\partial_y \overline{\theta}_s$ and the subtropical Δ_v vary substantially among simulations [*Schneider and Walker*, 2006], but they compensate in such a way that S_c remains roughly constant at the HC terminus. If Δ_v were evaluated in the deep tropics (where it is controlled by γ), the value of S_c at the HC terminus would then vary as Δ_h and/or γ change. This shows that variations in the subtropical static stability, rather than in the tropical static stability, control the HC extent.

4. Comparison with Alternative Formulations

[14] Some recent papers [e.g., *Lu et al.*, 2007; *Frierson et al.*, 2007] have advocated using the expressions for the HC terminus discussed by *Held* [2000], in which angular momentum conservation of the tropical upper-tropospheric flow and a measure of baroclinic instability are combined. We can derive an analogous expression using the result from the previous section. If the HC terminus were determined by the latitude to which upper-tropospheric angular momentum-conserving flow extends before the resulting zonal-wind shear is so large that $S_c > c$, a solution for the HC terminus can be obtained by assuming the vertically averaged temperature gradient in the troposphere scales with the near-surface potential temperature gradient $\partial_y \overline{\theta}_s$ and the zonal wind at the surface is negligible. Then using the small-angle approximation, the HC extends up to

$$\phi_c = \left(\frac{bc}{2} \frac{gH_t}{\Omega^2 a^2} \frac{\Delta_v}{T_0} \right)^{1/4}, \qquad (3)$$

where T_0 is a reference temperature, and b is an empirical constant to convert the scaling relation into an equality [cf. *Schneider*, 2006].

[15] For (3) to be a closed expression for the HC terminus within the context of axisymmetric, angular momentum–conserving HC theories, the height of the tropopause H_t and bulk stability Δ_v need to be evaluated in the deep tropics, at a latitude well removed from the modifying influences of baroclinic eddies. We calculated the expected terminus according to (3) by evaluating H_t and Δ_v averaged over $|\phi| \leq 5°$, and show the separation between it and the actual HC terminus in Figure 2a. (The constant $(bc/2)^{1/4} = 1.00$ is chosen such that (3) gives the actual latitude of the HC terminus in the mean over all simulations with $\gamma < 1$ and $\Delta_h \geq 60$ K.) The actual terminus shows little variation with Δ_h (Figure 1a), but the value expected according to (3) grows as Δ_v and H_t increase with increasing Δ_h. (The

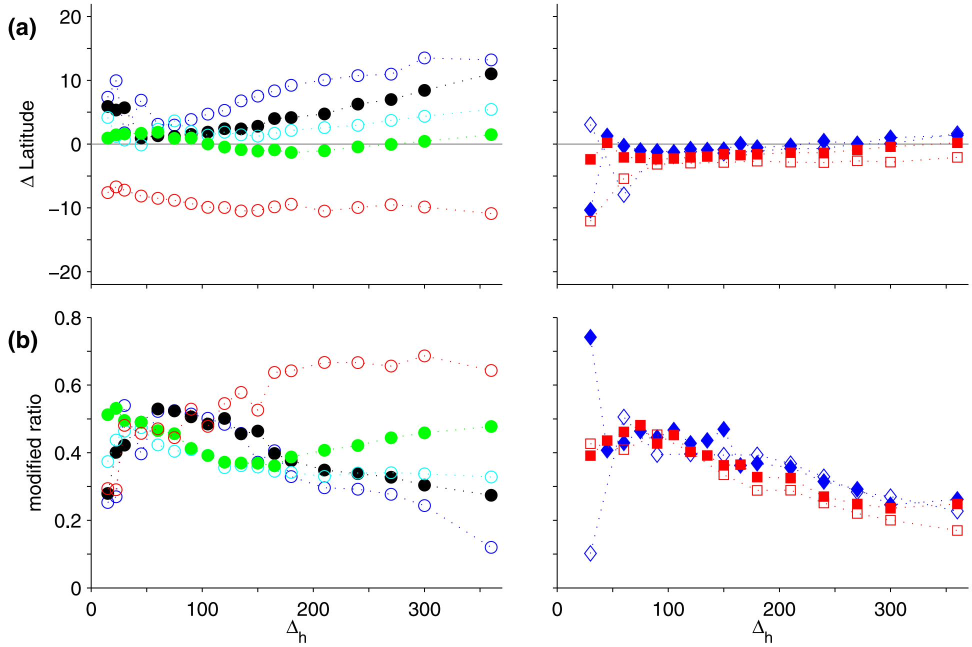

Figure 2. (a) As in Figure 1c, but showing difference between HC terminus expected according to (3) and actual terminus. (b) As in Figure 1b, but using a modified, quasi-geostrophic analog of (1); see text.

mean surface temperature—and with it Δ_v and H_t—increase in these simulations with increasing Δ_h [*Walker and Schneider*, 2006].) As inferred by *Walker and Schneider* [2006], the expected terminus according to (3) does not capture the variations with γ of the actual terminus. The separation between the two is no longer independent of Δ_h or γ. If (3) were heuristically modified so that H_t and Δ_v are evaluated in the subtropics near the HC terminus, the prediction performs no better than that shown here; such an expression captures the dependence on subtropical stability qualitatively, but quantitatively the fit is poor over the full range of simulations.

[16] Expression (3) has a form similar to the expression *Held* [2000] derived by combining the zonal wind from an angular momentum-conserving flow in the upper troposphere with the critical shear for baroclinic instability in the quasigeostrophic two-layer model. As discussed in the Introduction, however, there are important differences in interpretation. The derivation of (3) does not require a critical shear for baroclinic instability, which is an artifact of the two-layer model. Nonetheless, the difficulties arising from assuming upper-tropospheric flows strictly conserve angular momentum remain and account for the differences between the actual extent and that predicted by (3).

[17] Although the necessary condition for instability in the quasigeostrophic two-layer model formally resembles the condition $S_c \gtrsim 1$, the differences between the two conditions are significant. In the quasigeostrophic analog of (1), the meridional θ gradient is evaluated in the mid-troposphere, and a tropospheric mean value is substituted for the bulk stability Δ_v. With the meridional θ gradient evaluated as an average between $\sigma = 0.45$ and 0.55 and with $\overline{\theta}_t$ -$\overline{\theta}_s$ substituted for the bulk stability, the quasigeostrophic analog of (1) at the HC terminus is shown in Figure 2b. Compared to the nearly-constant critical value that (1) assumes at the HC terminus, the quasigeostrophic analog has considerably more scatter and does not assume a constant value. This is consistent with the findings of *Schneider and Walker* [2006] and their discussion of the differences between the bulk stability (2) and $\overline{\theta}_t$ - $\overline{\theta}_s$. If one goes further and combines the quasigeostrophic analog of (1) with the assumption of angular momentum–conserving upper-tropospheric flow to obtain the analog of (3) discussed by *Held* [2000], the fit to the simulation results degrades further, with a growing separation between actual and expected HC terminus emerging in each series of simulations as Δ_h increases. The upper-tropospheric flow in almost all of the simulations considered here is not close to angular momentum conserving.

5. Discussion

[18] We propose to take as the defining characteristic of the subtropical HC terminus that the divergence of eddy angular momentum fluxes in the upper troposphere changes sign there, as a result of vertical wave activity fluxes poleward of the terminus becoming sufficiently deep to reach the upper troposphere. Wave activity reaching the upper troposphere propagates meridionally and leads to eddy angular momentum flux convergence poleward of the HC and eddy angular momentum flux divergence within it, associated with mean meridional mass fluxes that are poleward in the upper branches of the Hadley cells and equatorward in the upper branches of the Ferrel cells. In a dry atmosphere, the supercriticality S_c as a measure of the depth of vertical wave activity fluxes, or meridional eddy entropy fluxes, relative to the tropopause height then gives an adequate measure of HC extent: the HC extends up to the latitude at which S_c first exceeds a critical values, which in our simulations is ~0.6. The supercriticality depends on both the subtropical static stability and meridional near-surface temperature gradient, and both vary substantially in

the simulations. In particular, the static stability is not merely convectively controlled but is influenced by large-scale dynamics, which means that theories for the subtropical static stability and temperature gradient are needed for a closed theory of the HC extent.

[19] A closed theory for the HC extent generally cannot be obtained by combining a criterion such as $S_c = c$ with the assumption that the upper-tropospheric flow in the HC is angular momentum conserving. The resulting expressions such as (3) give poor fits to our simulation results of a wide range of climates. Moreover, although the supercriticality (1) has a functional form similar to the supercriticality that controls the degree of instability in the quasigeostrophic two-layer model [*Phillips*, 1954], adopting the latter yields poorer fits to the simulation results. For the narrower range of climates occupied by Earth, the quantitative distinction between our criterion based on (1) and a heuristically modified version of (3) may be small. But our interpretation that the HC extends up to the latitude at which vertical fluxes of wave activity reach the upper troposphere provides a more general foundation for the relation of baroclinic eddies and HC extent.

[20] We expect the general result—the HC extends to the latitude at which substantial vertical wave activity fluxes reach the upper troposphere—to carry over to sufficiently baroclinic moist atmospheres. But a challenge posed by the moist problem is identifying the appropriate moist bulk stability that should be used in a supercriticality measure such that it becomes an adequate measure of the depth over which vertical wave activity fluxes extend; in moist atmospheres, the supercriticality (1) is not generally an adequate measure of the depth over which vertical wave activity fluxes or meridional eddy entropy fluxes extend [*Schneider and O'Gorman*, 2008].

[21] These complications notwithstanding, our results suggest that a subtropical static stability measure plays an important role in controlling the HC extent. To the degree that an increased dry static stability implies an increased effective moist static stability and hence a decreased depth of vertical wave activity fluxes, they imply that the HC terminus can be expected to shift poleward as the subtropical static stability increases and/or the meridional near-surface temperature gradient decreases. Qualitatively, this is consistent with what is seen in simulations of the 21st-century climate. To obtain a quantitative theory for moist atmospheres, it will be necessary to understand what controls the relevant subtropical static stability in moist atmospheres.

[22] **Acknowledgments.** This work was supported by a David and Lucile Packard Fellowship and by the National Science Foundation (grant ATM-0450059). Three anonymous reviewers provided very helpful comments.

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