Response of idealized Hadley circulations to seasonally varying heating

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[1] The response of Hadley circulations to displacements of the latitude of maximum heating is investigated in idealized axisymmetric and eddy-permitting models. Consistent with an earlier study and with theory for the nearly inviscid limit (Lindzen and Hou, 1988), the strength of the Hadley circulation is sensitive to displacements of heating: the winter cell strengthens and summer cell weakens when the maximum heating is displaced off the equator. However, in conflict with the nearly inviscid limit but consistent with observations of Earth's atmosphere, the strength of an annually averaged Hadley circulation is comparable to the Hadley circulation driven by an annually averaged heating. The disagreement between these results and the nearly inviscid limit is ascribed to vertical diffusion of momentum and dry static energy in the axisymmetric model and to baroclinic eddy fluxes in the eddy-permitting model. Nonlinear amplification of the annually averaged Hadley circulation is only seen near the upper boundary in simulations with a rigid lid near the tropopause, suggesting that the amplification is an artifact of the upper boundary condition. **Citation:** Walker, C. C., and T. Schneider (2005), Response of idealized Hadley circulations to seasonally varying heating, *Geophys. Res. Lett.*, *32*, L06813, doi:10.1029/2004GL022304.

1. Introduction

[2] The Hadley circulation in idealized axisymmetric models driven by equinoctial heating is much weaker than the annually averaged Hadley circulation of Earth's atmosphere [e.g., *Held and Hou*, 1980]. *Lindzen and Hou* [1988] (hereinafter referred to as LH) attempted to explain this discrepancy in a study of the Hadley circulation's response to heating with a maximum displaced off the equator, in which they found that the strength of the Hadley circulation in axisymmetric models depends sensitively on the latitude of maximum heating. This led LH to propose that the annually averaged Hadley circulation is primarily an average of solstitial circulations, that the Hadley circulation 'flips' between these solstitial states, and that these flips manifest themselves as a square-wave dependence of Hadley circulation strength on time. LH suggested that the annually averaged Hadley circulation is much stronger than the Hadley circulation driven by annually averaged heating, implying a nonlinear amplification of the annually averaged response to seasonally varying heating.

[3] LH arrived at these conclusions by applying arguments posited by *Held and Hou* [1980] for the "nearly inviscid" limit of vanishing viscosity: that the flow in the upper branch of the Hadley circulation conserves angular momentum and that the Hadley circulation is energetically closed, so that heating in the ascending branch of each cell is balanced by cooling in the descending branch. These constraints, together with the assumption of a balanced zonal wind and with the representation of heating as Newtonian relaxation toward a radiative equilibrium state, allowed LH to derive estimates of Hadley circulation strength as a function of the latitude of maximum radiative equilibrium potential temperature. Figure 1a shows such estimates for four values of the latitude of maximum radiative equilibrium potential temperature, ϕ_0, from a maximum at the equator (dotted line) to a 6° displacement into the northern hemisphere (solid line). Even slight variations in ϕ_0 dramatically increase the southern (winter) cell and decrease the northern (summer) cell.

[4] A strengthening of the winter cell does not necessarily imply nonlinear amplification of the annually averaged Hadley circulation, since a strengthening of the winter cell may be compensated by a weakening of the summer cell in such a way that the annually averaged circulation is close to the circulation driven by annually averaged heating. Figure 1b, the average of the circulations in Figure 1a and of their southern analogs (e.g., the average of the circulations with $\phi_0 = 6°$ and $\phi_0 = -6°$), shows that there is some compensation between winter and summer cells for off-equatorial heating, as the mass flux maximum of the averaged circulations in Figure 1b is slightly less than half, and slightly poleward of, the mass flux maximum of the corresponding winter cells. Nonetheless, the averaged circulations are substantially stronger — by a factor of 15 for $\phi_0 = 6°$ — than the circulations driven by heating with $\phi_0 = 0$.

[5] *Dima and Wallace* [2003] recently examined the seasonality of Earth's Hadley circulation and, in conflict with LH, did not see evidence of nonlinear amplification of the annual average circulation. They found that, throughout the seasonal cycle, a significant fraction of the Hadley circulation can be attributed to an equinoctial component that is roughly symmetric about the mean latitude of the intertropical convergence zone. The amplitude of this equinoctial component varies sinusoidally with season and does not exhibit the dramatic equinox–solstice swings implied by Figure 1, instead staying within about 40% of its annually averaged value. Variation of the Hadley circulation with season is due primarily to a seasonally reversing solstitial component, which likewise varies sinusoidally and is largely associated with monsoons.

[6] One mechanism that might prevent nonlinear amplification of the annual average circulation and might explain the difference between LH and observations was explored

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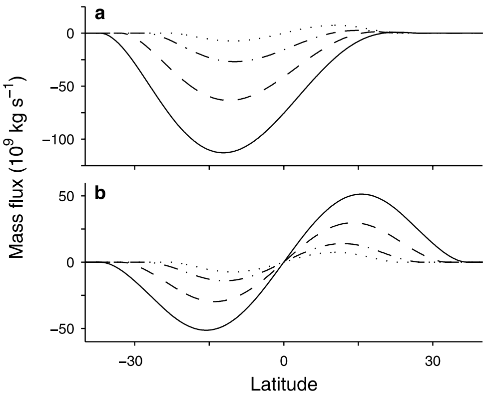

Figure 1. Mass flux (a) and average (e.g., $\phi_0 = 6°$ with $\phi_0 = -6°$) mass flux (b) in the upper branches of nearly inviscid axisymmetric Hadley circulations with latitude of maximum radiative equilibrium potential temperature $\phi_0 = 0$ (dotted), 2° (dash-dotted), 4° (dashed), and 6° (solid). Positive values indicate northward flux. Mass fluxes are calculated with a radiative equilibrium potential temperature that varies with latitude ϕ as $\theta_{eq}(\phi) = 315$ K $- \Delta_h(\sin^2\phi - 2\sin\phi_0\sin\phi)$, where $\Delta_h = 60$ K. The thermal relaxation time $\tau = 30$ days, domain height $H = 12$ km, reference density $\rho_0 = 1$ kg m^{-3}, and gross stability 30 K (effective potential temperature difference between upper and lower branches of Hadley circulation) are chosen to be roughly consistent with the simulations in Figures 2a and 2b. See LH and *Held and Hou* [1980] for other parameters and details of the calculation.

by *Fang and Tung* [1999], who noted that the axisymmetric model used by LH takes longer than a season to equilibrate so that the average of steady-state circulations might not be representative of the annually averaged Hadley circulation. Using a model with sinusoidally varying heating, *Fang and Tung* [1999] found that nonstationarity decreases nonlinear amplification of the annually averaged Hadley circulation only marginally but that the strength of the Hadley circulation varies sinusoidally in time.

[7] To reconcile the differences between the modeling studies and *Dima and Wallace* [2003], we consider the roles of baroclinic eddies and vertical diffusion in damping the nonlinear amplification of the annually averaged Hadley circulation.

2. Model

[8] We conducted simulations with a GCM that includes idealized representations of radiative forcing and turbulent dissipation in the planetary boundary layer. The model surface is thermally insulating and spatially uniform, with no topography and a constant roughness length. The model is a spectral-transform model, run in eddy-permitting and axisymmetric configurations, in both cases with T85 horizontal resolution and 30 unequally spaced sigma levels in the vertical (the axisymmetric configuration is truncated at zonal wave number zero).

[9] Radiative forcing is represented as Newtonian relaxation of temperatures toward a zonally symmetric, statically stable radiative equilibrium state, with a constant relaxation timescale of 30 days. The radiative equilibrium temperature is given by $T_{eq} = \max[200\text{ K}, T^*_{eq}(p, \phi)]$ with

$$T^*_{eq}(p,\phi) = \left[315\,\mathrm{K} - \Delta_h\left(\sin^2\phi - 2\sin\phi_0\sin\phi\right) - \Delta_v \log\left(\frac{p}{p_0}\right)\cos^2(\phi-\phi_0)\right]\left(\frac{p}{p_0}\right)^{\kappa},$$

where $\Delta_h = 60$ K is a meridional temperature difference, $\Delta_v = 10$ K a static stability parameter, and $p_0 = 1000$ hPa a reference pressure. This radiative equilibrium temperature is identical to that of *Held and Suarez* [1994], except that the additional parameter ϕ_0 allows the equilibrium temperature maximum to be shifted off the equator. We show simulations with $\phi_0 = 0$ and 6°. LH use a radiative equilibrium temperature that has a similar dependence on latitude, but with a vertical structure that is more stable at the equator and with a rigid lid in place of an isothermal upper atmosphere.

[10] Turbulent dissipation in a planetary boundary layer of fixed height (2.5 km) is represented as vertical diffusion of momentum and dry static energy [*Smagorinsky et al.*, 1965]. In the eddy-permitting simulations, aside from Newtonian relaxation of temperatures, small-scale horizontal hyperdiffusion is the only dissipative process above the planetary boundary layer. In the axisymmetric simulations, we included vertical diffusion of momentum and dry static energy above the boundary layer, with Prandtl number one and with constant diffusivity 5 m^2 s^{-1}. This diffusivity is near the minimum diffusivity for which we were able to obtain steady states in the simulations with $\phi_0 = 6°$. It is equal or comparable to diffusivities used by LH and *Fang and Tung* [1999].

3. Response to Off-Equatorial Heating

[11] Here we show axisymmetric and eddy-permitting simulations with a radiative equilibrium temperature symmetric about the equator ($\phi_0 = 0$), mimicking annually averaged heating, and with a radiative equilibrium temperature with a maximum displaced off the equator ($\phi_0 = 6°$), mimicking southern hemisphere winter. The axisymmetric results are steady-state solutions; the eddy-permitting results are averages over 200 days of simulations in statistically steady states.

3.1. Axisymmetric Simulations

[12] Consistent with LH, our axisymmetric simulations show significant changes in Hadley circulation strength when the heating is displaced off the equator. For $\phi_0 = 6°$, the strength (maximum absolute value of the stream function) of the winter cell increases by approximately a factor of three; the summer cell decreases to one-fourth the value for $\phi_0 = 0$ (Figure 2b). Both changes are similar to those shown by LH for the same displacement of the equilibrium temperature maximum (see LH, Figure 9).

[13] Although the changes in strength are similar to those in LH, there are structural differences between the two sets of circulations. Each cell of the circulations in Figures 2a and 2b has a single maximum above the top of the boundary layer ($\sigma \approx 0.8$). In contrast, the circulation cells shown by LH have two maxima, one near the surface and the other near the upper boundary. The cause and implications of the high-altitude maxima are discussed in section 4.

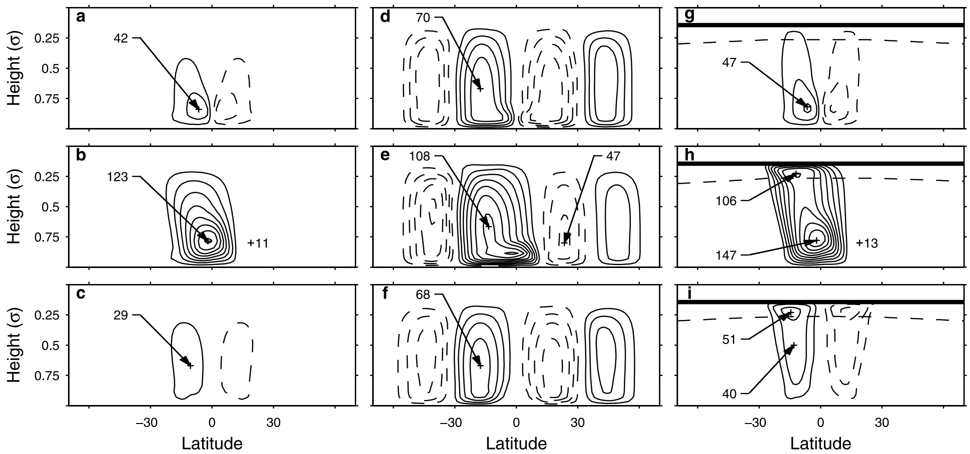

Figure 2. Mass flux stream functions in axisymmetric (left column) and eddy-permitting (middle column) simulations, and in axisymmetric simulations with a rigid lid at $\sigma = 0.15$ (right column). Top row: $\phi_0 = 0$. Middle row: $\phi_0 = 6°$. Bottom row: average of $\phi_0 = 6°$ and $\phi_0 = -6°$. Contour interval is 15×10^9 kg s^{-1}. Dashed contours correspond to negative values (clockwise rotation), solid contours to positive values (counterclockwise rotation). In the right column, the thick line indicates the rigid lid and the dashed line demarcates a boundary layer. Maxima above $\sigma = 0.85$ are identified.

[14] Changes in the location and extent of the Hadley circulation mirror those documented by LH. In the simulation with radiative equilibrium temperature symmetric about the equator (Figure 2a), the latitude separating the two circulation cells and the latitude of maximum ascent are coincident at $\phi_0 = 0$. When the radiative equilibrium temperature maximum is displaced off the equator (Figure 2b), the separation between the summer and winter cells is no longer at $\phi_0 = 6°$ but is shifted to approximately 10°. The latitude of maximum ascent remains closer to ϕ_0. As discussed by LH, the winter cell expands and the summer cell contracts.

[15] LH concluded that the average of the Hadley circulations obtained for $\phi_0 = 6°$ and $\phi_0 = -6°$, mimicking an 'annually averaged' circulation, is substantially stronger than the Hadley circulation for $\phi_0 = 0$, mimicking a circulation driven by 'annually averaged' heating. Figure 2c shows the average of the $\phi_0 = 6°$ and $-6°$ circulations. Comparison of Figures 2c and 2a reveals that in our model, in contrast with LH, the 'annually averaged' Hadley circulation is slightly *weaker* than the Hadley circulation driven by 'annually averaged' heating. LH do not show a figure analogous to Figure 2c, instead pointing to the relative strength of summer and winter cells in their $\phi_0 = 6°$ case as evidence of nonlinear amplification of the annually averaged Hadley circulation.

[16] As in LH, the simulations in Figures 2a and 2b become symmetrically unstable when the diffusivity is reduced to 2.5 m^2 s^{-1} or lower. When the diffusivity is increased — we carried out simulations with diffusivities up to 10 m^2 s^{-1} — the strength of the circulations increases, but the results do not change qualitatively.

3.2. Eddy-Permitting Simulations

[17] Figures 2d and 2e show the mass flux stream functions for eddy-permitting simulations with $\phi_0 = 0$ and 6°. Although the relative changes are smaller than in the axisymmetric simulations, the strength of the circulation again reacts to latitudinal displacements of the radiative equilibrium temperature maximum. For $\phi = 6°$, the strength of the winter cell increases by a factor of 1.5 compared with the simulation with $\phi_0 = 0$; the strength of the summer cell decreases by the same factor. Eddies strengthen the Hadley circulation for annually averaged heating (compare Figures 2a and 2d), while they weaken the winter cell (Figures 2b and 2e). The strength of the summer and winter cell is of the same order of magnitude as that of the observed Hadley cells. Eddies also lead to a poleward expansion of the Hadley circulation.

[18] Even more so than in the axisymmetric simulations, the average of the $\phi_0 = 6°$ and $-6°$ circulations (Figure 2f) is similar to the circulation for $\phi_0 = 0$ (Figure 2d), indicating that, in the presence of eddies, the Hadley circulation does not undergo nonlinear amplification of its annual average.

4. Effects of Rigid Lid

[19] In both axisymmetric and eddy-permitting simulations, the Hadley circulation for $\phi_0 = 0$ is of similar strength as the average of the circulations for $\phi_0 = 6°$ and $-6°$. Using an axisymmetric model similar to ours, LH reached the contradictory conclusion that the average of the Hadley circulations for $\phi_0 = 6°$ and $-6°$ is nonlinearly amplified compared with the circulation for $\phi_0 = 0$. Here we suggest that the amplification described by LH is spurious, caused by the rigid lid used in their model.

[20] Figures 2g and 2h show the mass flux stream functions in our axisymmetric model when a stress-free rigid lid is imposed at $\sigma = 0.15$. The effect of the rigid lid is manifest when these figures are contrasted with Figures 2a and 2b, which differ only in that the rigid lid is at $\sigma = 0$, far removed from the upper branch of the Hadley circulation (and at an altitude at which the radiative equilibrium temperature is constant). The rigid lid amplifies the circulation near the upper boundary, especially in the winter cell in the simulation with $\phi_0 = 6°$, in which a second stream function maximum forms near $\sigma = 0.2$, similar to the upper-level maxima seen in LH and in other simulations [e.g., *Held and Hou*, 1980; *Fang and Tung*, 1999].

[21] The high-altitude amplification leads to a stream function maximum near the upper boundary when the ϕ_0 = 6° and −6° circulations are averaged (Figure 2i). When the averaged stream function is compared with the stream function for ϕ_0 = 0 in Figure 2g, it appears that there is nonlinear amplification near the upper boundary; below the upper maximum, the two stream functions are similar.

[22] The effect of the rigid lid can be understood by considering the inviscid balance relation for the zonal wind,

$$\frac{\partial}{\partial z}\left(fu + \frac{u^2 \tan\phi}{a}\right) \propto -\frac{g}{a}\frac{\partial\theta}{\partial\phi},$$

where θ is potential temperature. The no-stress condition dictates $\partial u/\partial z = 0$ at the upper boundary, which, if there is a meridional potential temperature gradient there — for example, due to a meridional gradient in radiative equilibrium temperature — makes inviscidly balanced flow unrealizable as a solution to the equations of motion. A boundary layer forms near the rigid lid in which the flow makes a transition from the nearly inviscidly balanced flow in the interior to the stress-free boundary [cf. *Fang and Tung*, 1994].

[23] Scaling arguments show that the thickness of this boundary layer is of order $\sqrt{\nu\tau}$, where τ = 30 days is the thermal relaxation time and $\nu = 5$ m^2 s^{-1} the diffusivity [cf. *Fang and Tung*, 1996]. A boundary layer of this thickness is demarcated by the dashed line in Figures 2g–2h. The nonlinear amplification near the upper boundary in Figure 2i coincides with the boundary layer, leading us to conclude that this amplification is an artifact of the upper boundary condition. Rigid-lid artifacts will also play a role in the nonlinear amplification of the averaged time-dependent Hadley circulations of *Fang and Tung* [1999].

5. Discussion and Conclusions

[24] The nonlinear amplification of annually averaged Hadley circulations in the nearly inviscid limit is predicated on the twin constraints of angular momentum conservation and energetically closed circulation cells. We have shown that vertical diffusion in axisymmetric simulations and baroclinic eddies in eddy-permitting simulations violate these constraints, preventing nonlinear amplification.

[25] In conflict with LH's simulations but consistent with observations [*Dima and Wallace*, 2003], we find that an 'annually averaged' axisymmetric Hadley circulation is close to the circulation forced by an 'annually averaged' heating. The discrepancy between our conclusions and those of LH can be attributed to the rigid lid in LH's model.

[26] Although the diffusivity in the axisymmetric simulations is near the minimum diffusivity with which we were able to obtain steady states for ϕ_0 = 6°, vertical diffusion of momentum and dry static energy accounts for the disagreement with the nearly inviscid limit. The diffusivity is small enough that the poleward flow over much of the upper branch of the winter cell for ϕ_0 = 6° approximately conserves angular momentum. The meridional gradient of the zonal wind is within 10% of the gradient of the angular momentum conserving zonal wind over much of the upper branch of the winter cell, except near the poleward edge of the cell, where zonal wind gradients deviate significantly from angular momentum conservation. Subtropical fronts with near-discontinuities in zonal wind form at the poleward edges of axisymmetric Hadley cells, and irrespective of the value of the diffusivity, vertical diffusion is important in the shear zones at these fronts [cf. *Held and Hou*, 1980; *Schneider*, 1984]. Vertical diffusion is also important in the summer cell (ϕ_0 = 6°) and in the equinoctial cells (ϕ_0 = 0), which all are weaker than the winter cell, thus increasing the effectiveness of diffusion relative to advection. Meridional gradients of zonal winds in the upper branches deviate from angular momentum conservation by more than 50% in the summer cell and by more than 25% in the equinoctial cells. The fact that, with smaller diffusivities, we were unable to obtain steady circulations for heating displaced off the equator suggests that, if diffusivities are taken to be constant, the nearly inviscid limit may not be simultaneously relevant for the winter, summer, and equinoctial cells. The dependence of the strength of the circulation cells on diffusivities [cf. *Held and Hou*, 1980], and the different degrees to which winter, summer, and equinoctial cells approach the nearly inviscid limit, invalidate the predictions of the nearly inviscid limit for their relative strengths.

[27] In the eddy-permitting simulations, the subtropical divergences of baroclinic eddy fluxes of momentum and heat have an effect similar to that of vertical diffusion of momentum and dry static energy in the axisymmetric simulations, raising the question of the degree to which baroclinic eddies may influence the strength of Hadley circulations.

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