Climate & the Global Circulation of the Atmosphere

Assignment class

7/12/15

Homeworks

- **Homework #5** will be returned at the last assignment class (Dec 14th)
- **Final homework:** To be released on Wednesday and due in the New Year - can ask questions at next Monday's class

Outline for today

- Hide's theorem and its implications for (maximum) winds on Earth
- The Hadley circulation and temperature gradients

Hide's theorem: *in steady state, there can be no extrema of angular momentum except at the lower boundary*

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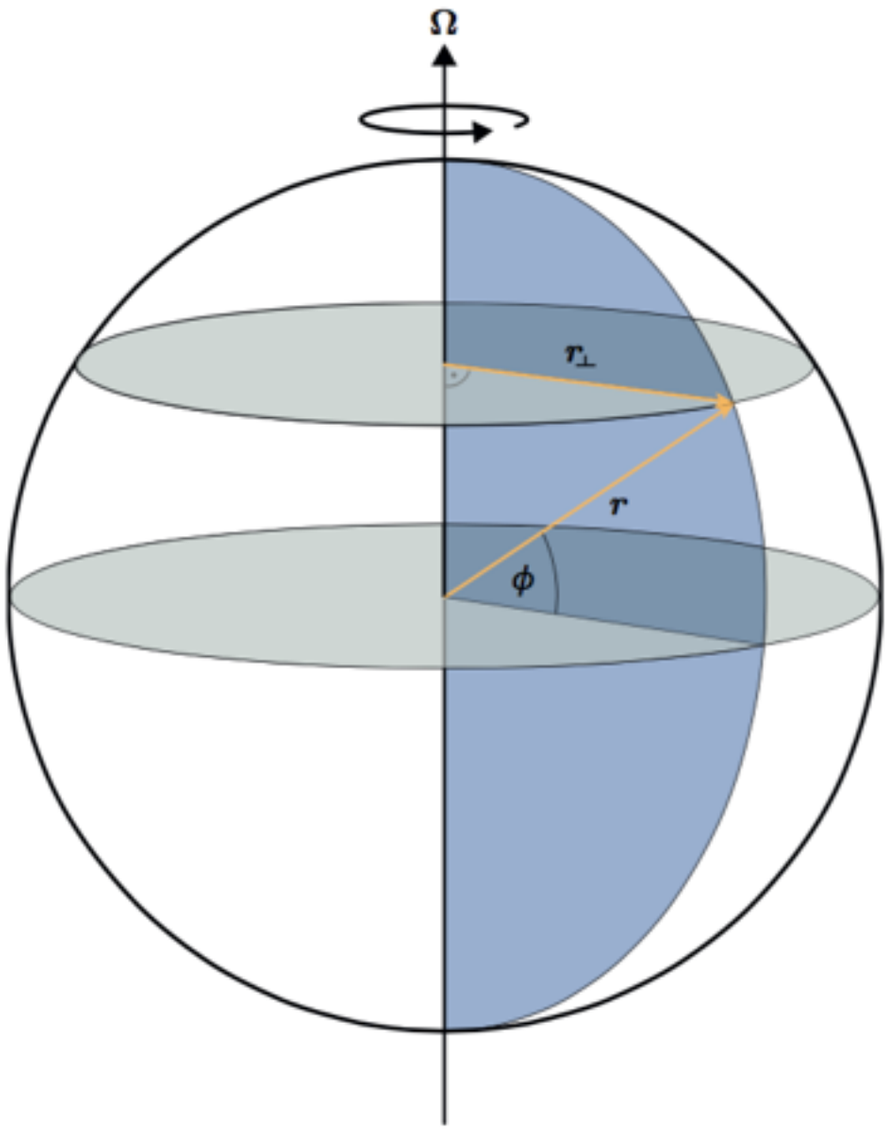
What does this tell us about the maximum possible winds on Earth?

AM definition

What is the angular momentum?

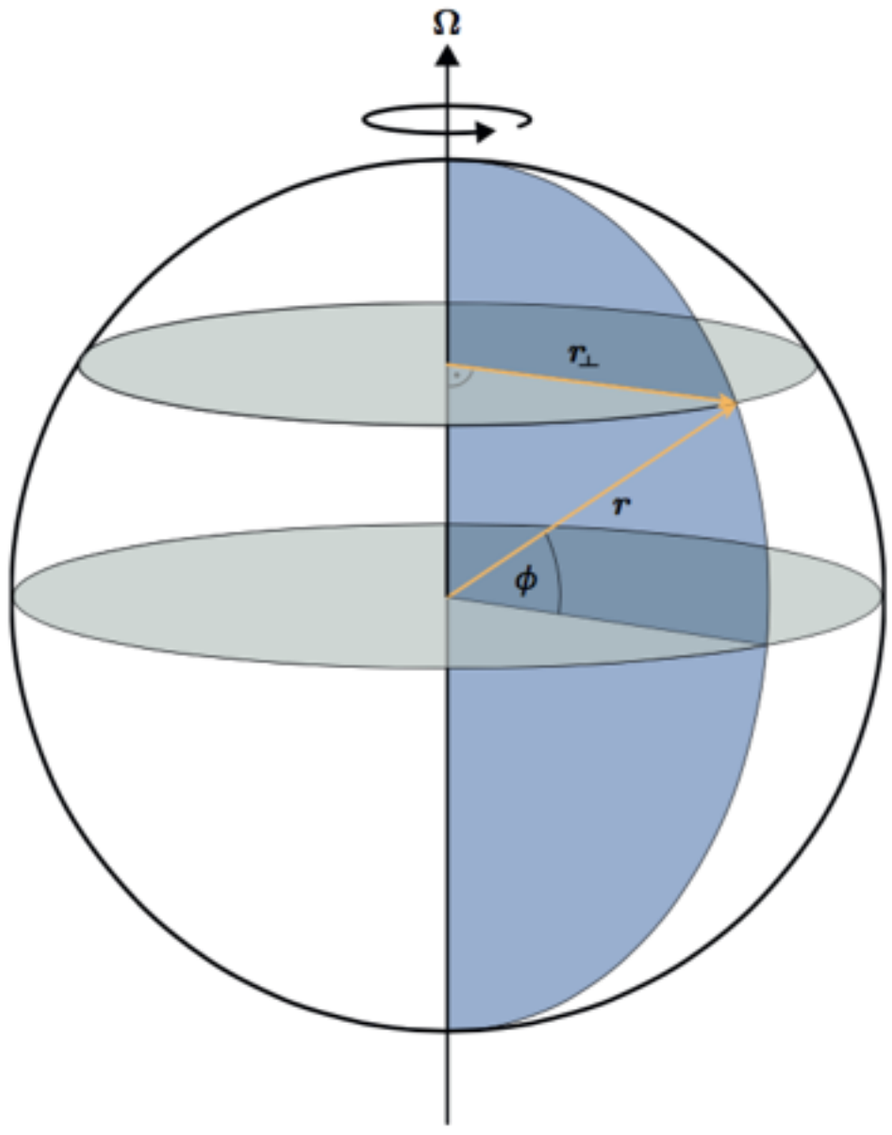


AM definition

What is the angular momentum?

$$M = (\Omega r_\perp + u) r_\perp$$

$$r_\perp = a \cos \phi$$



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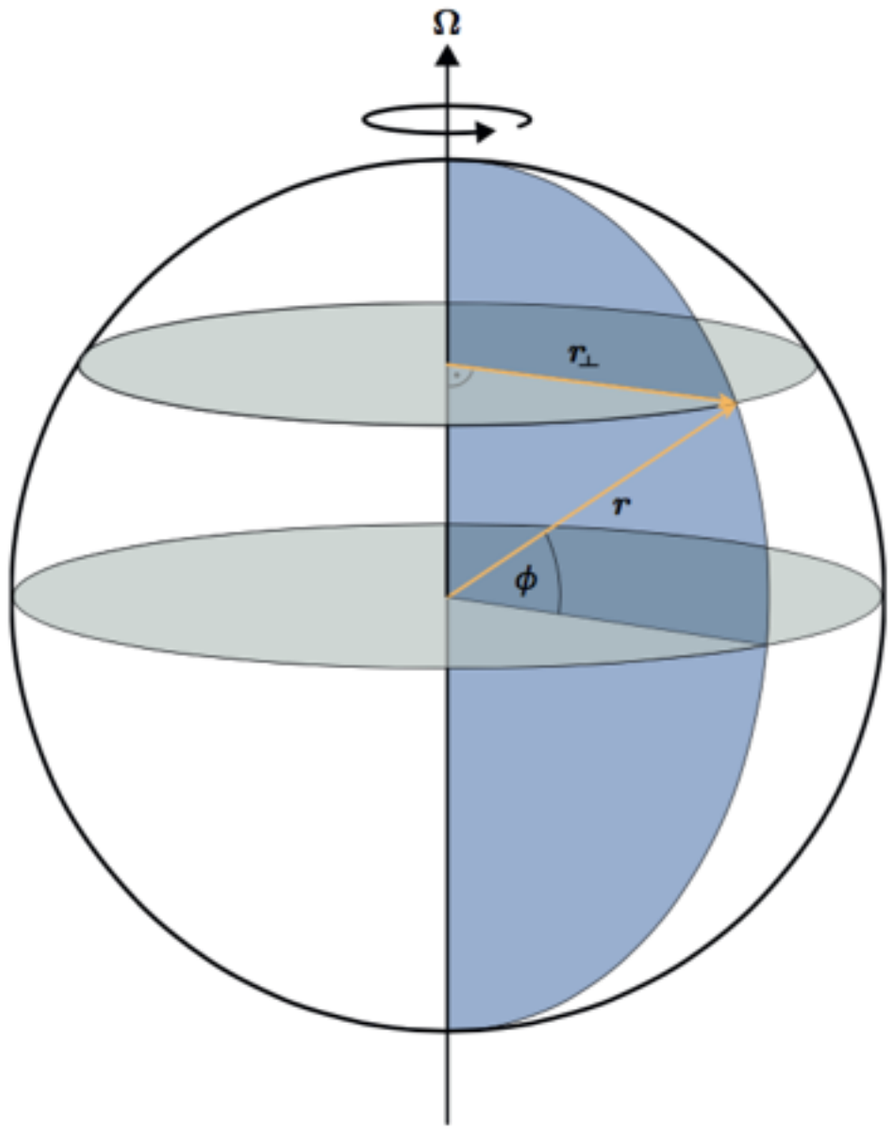

Where and what is the maximum angular momentum (assuming Hide's theorem is satisfied)?

$$M_{\max} = \text{???}$$

Maximum AM

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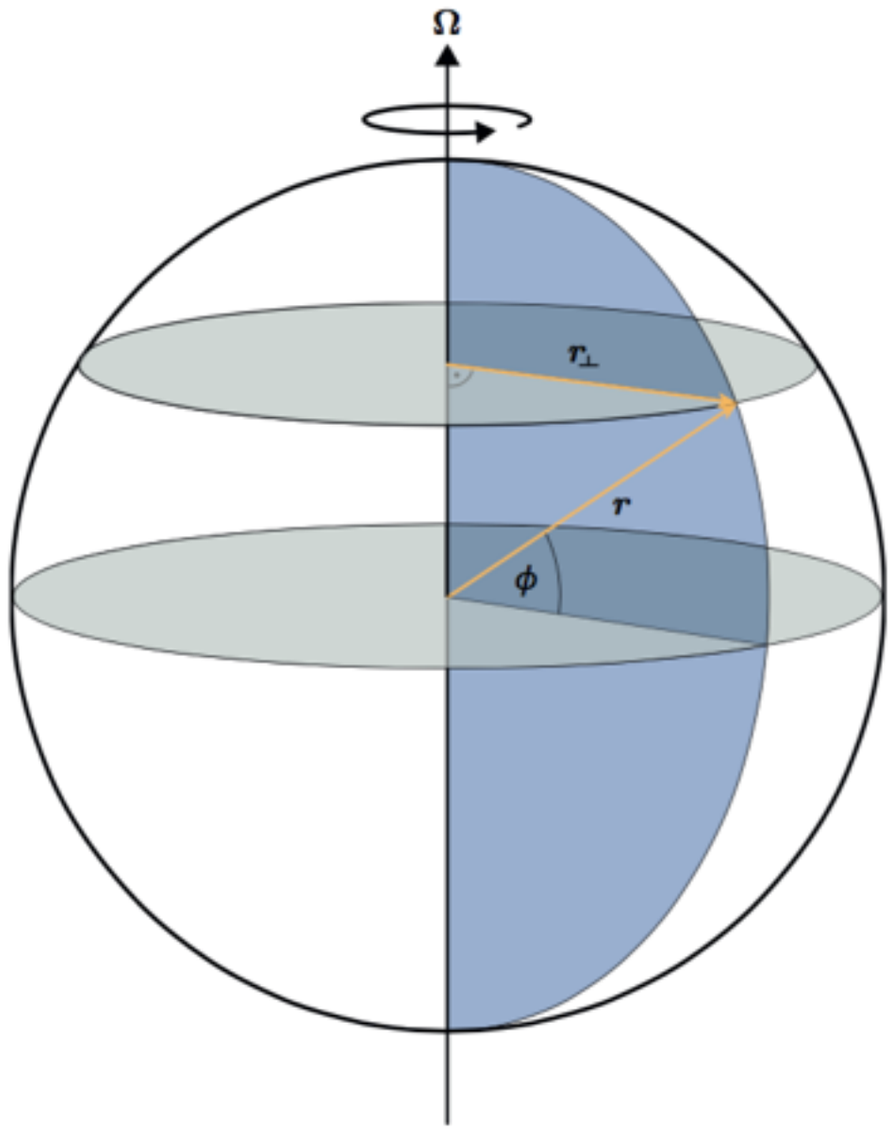
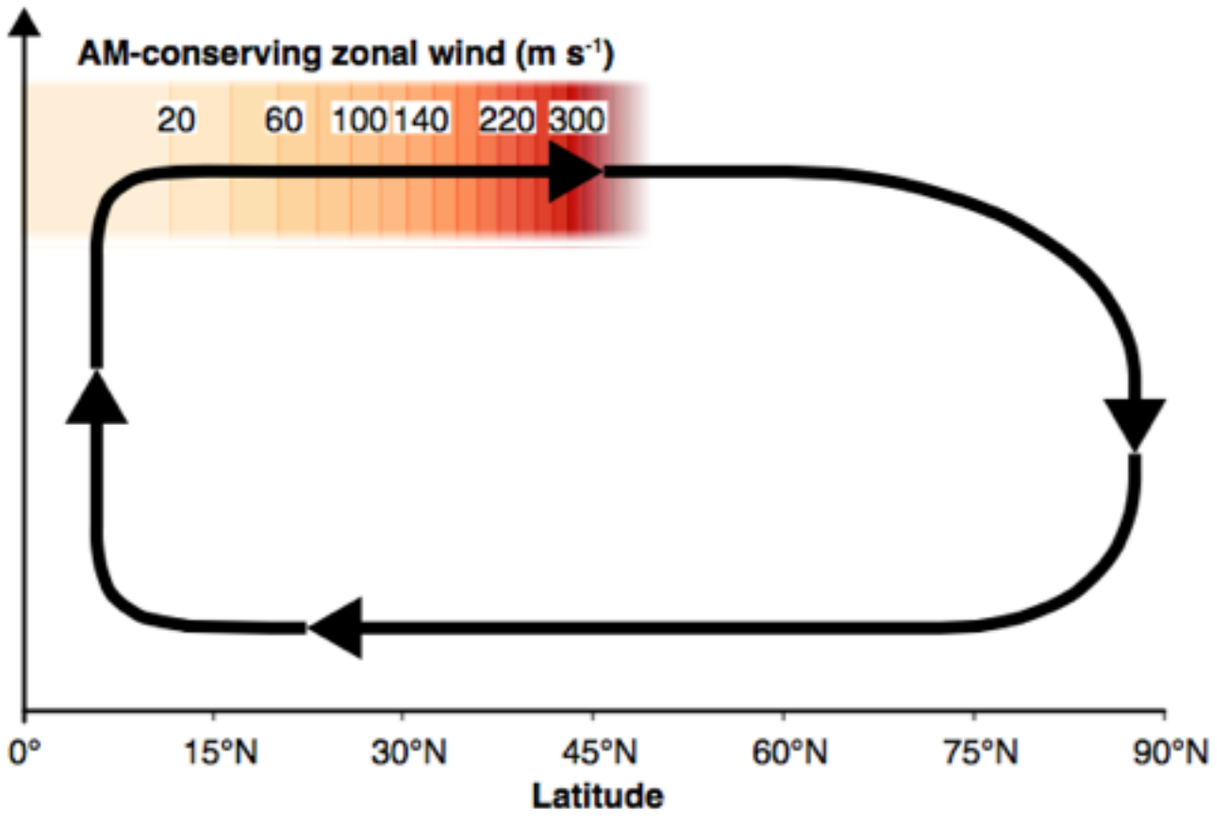


Where and what is the maximum angular momentum (assuming Hide's theorem is satisfied)?

$$M_{\max} = \Omega a^2$$

Maximum winds on Earth

Assuming air moves poleward from the Equator and conserved its AM - what will be the zonal wind speed?



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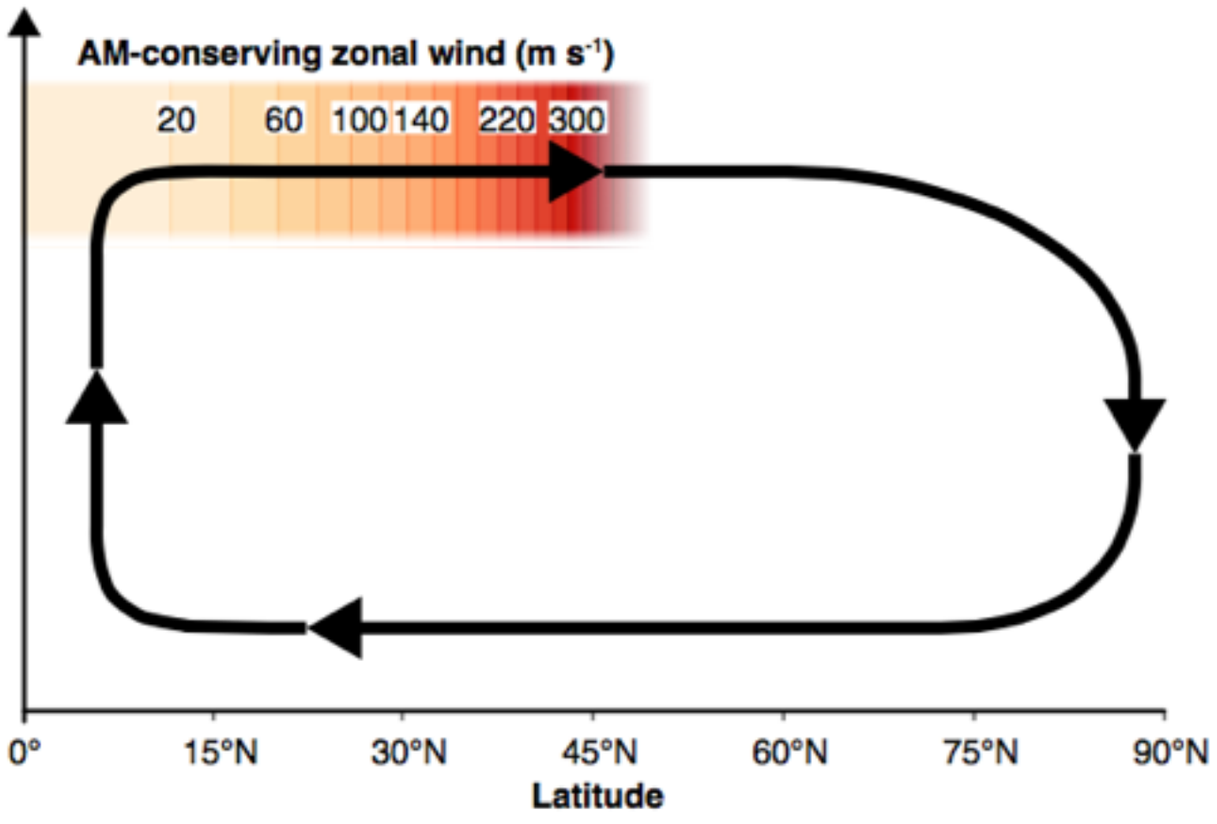

Maximum winds on Earth

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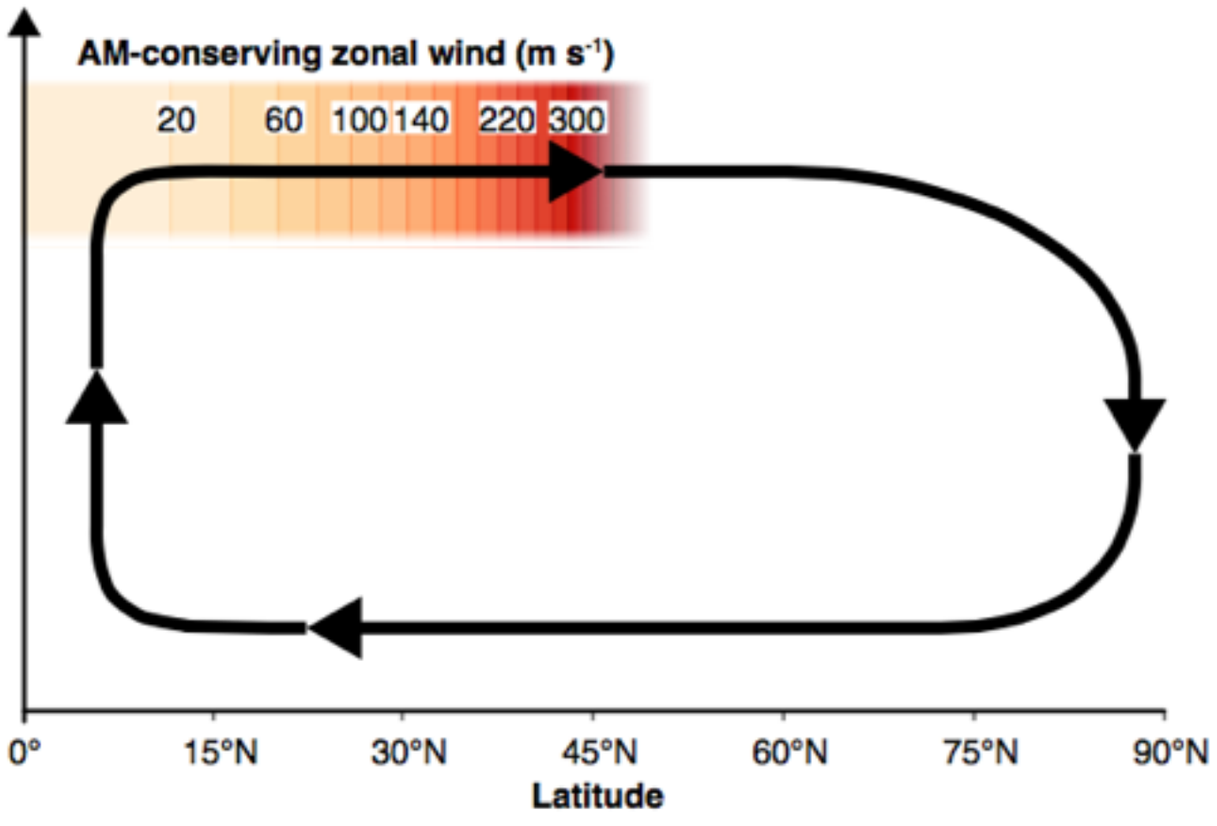
$$M = M_{\max} = \Omega a^2$$

$$\Omega a^2 \cos^2\phi + u_{\max} a \cos\phi = \Omega a^2$$

$$\Rightarrow u_{\max} = \Omega a \frac{\sin^2\phi}{\cos\phi}$$



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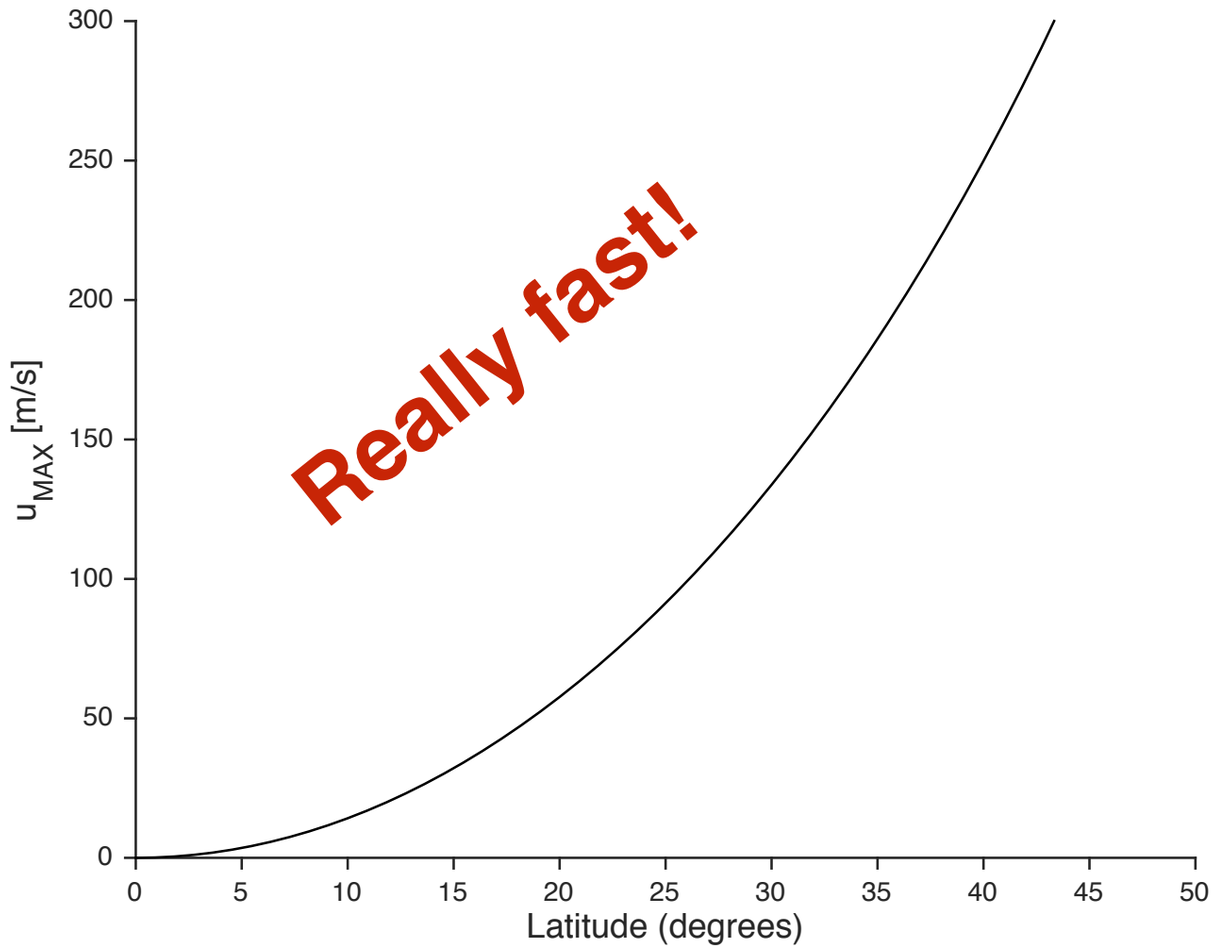
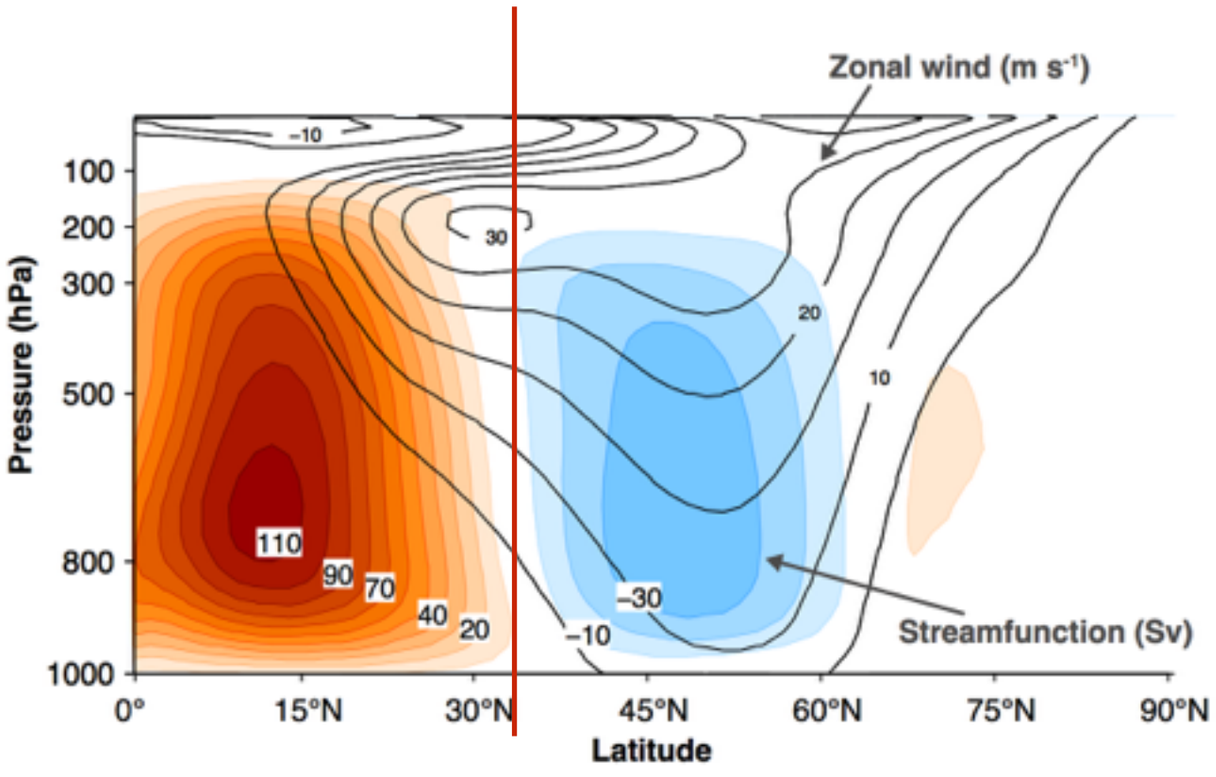


Winds, Hadley circulation, and temperature gradients

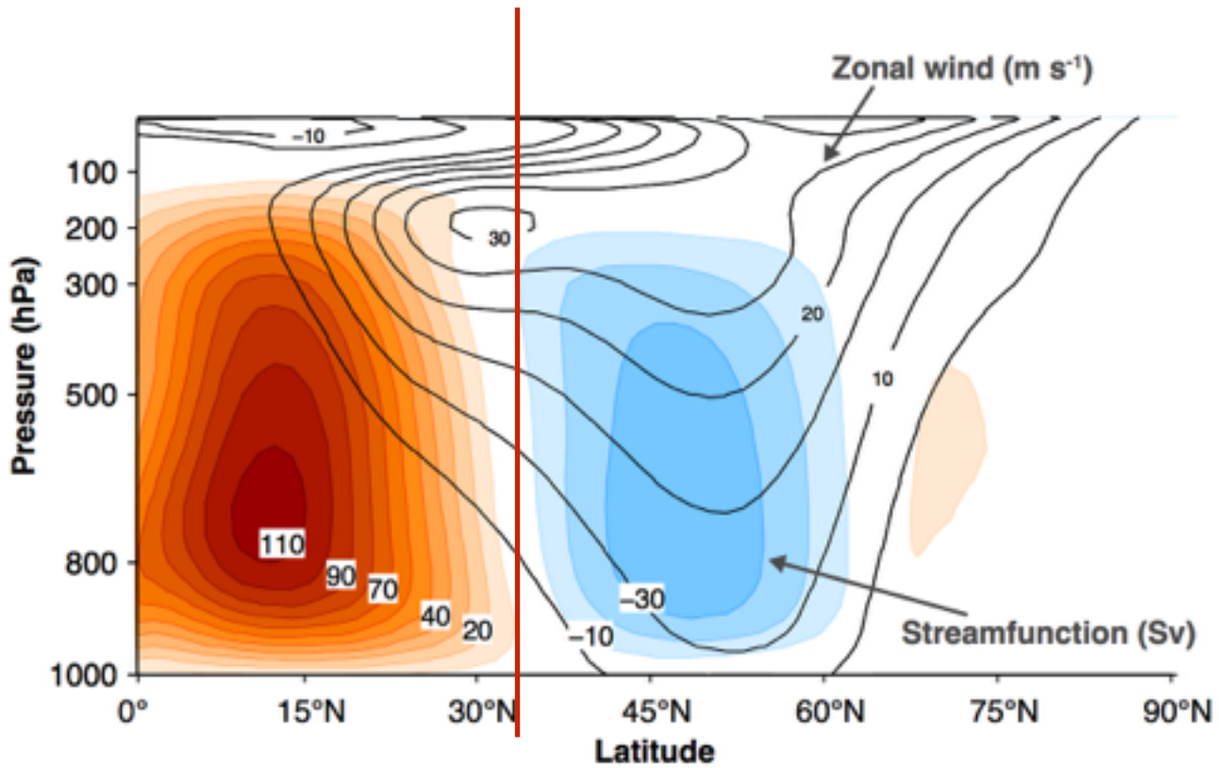
What does Hide's theorem imply about meridional temperature gradients and the extend of the Hadley cell?



Winds, Hadley circulation, and temperature gradients

Relate zonal winds in the Hadley circulation to pressure and temperature gradients:

$$2u\Omega \sin\phi \approx -\frac{1}{a\rho}\frac{\partial p}{\partial \phi}$$



approximation to the meridional momentum equation, known as…?

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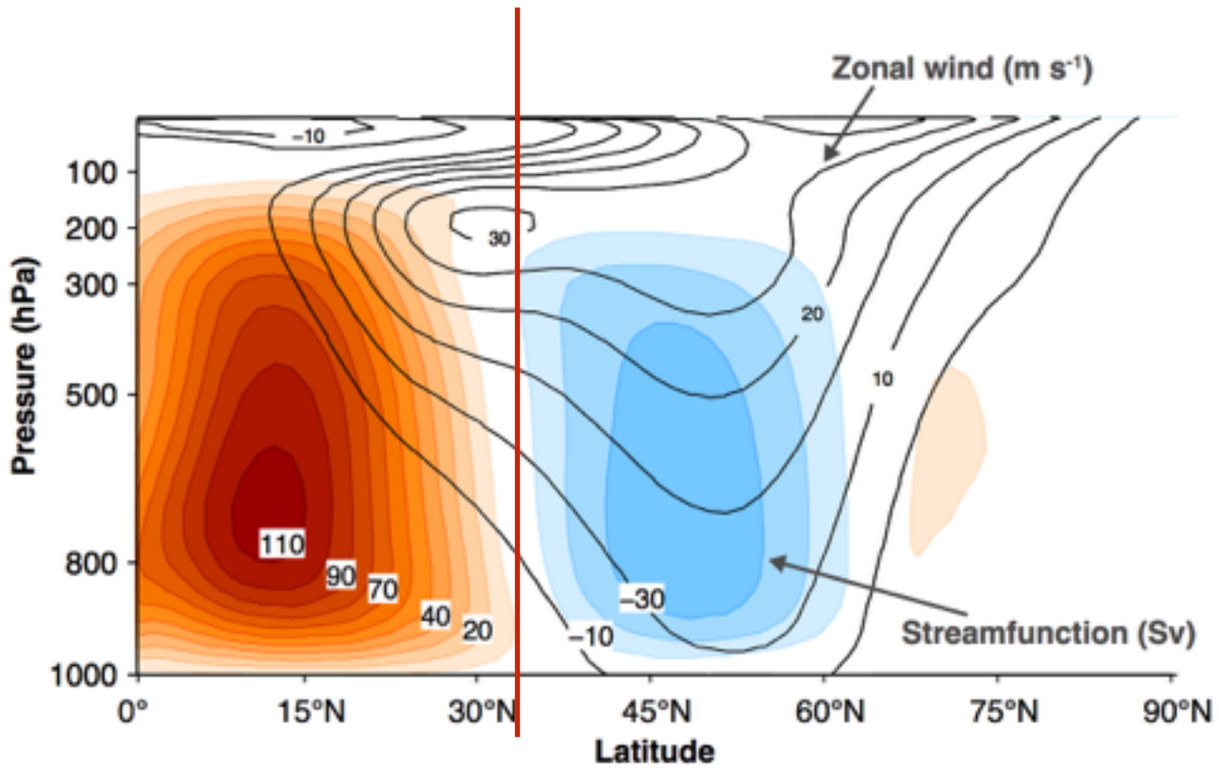
geostrophic balance



Winds, Hadley circulation, and temperature gradients

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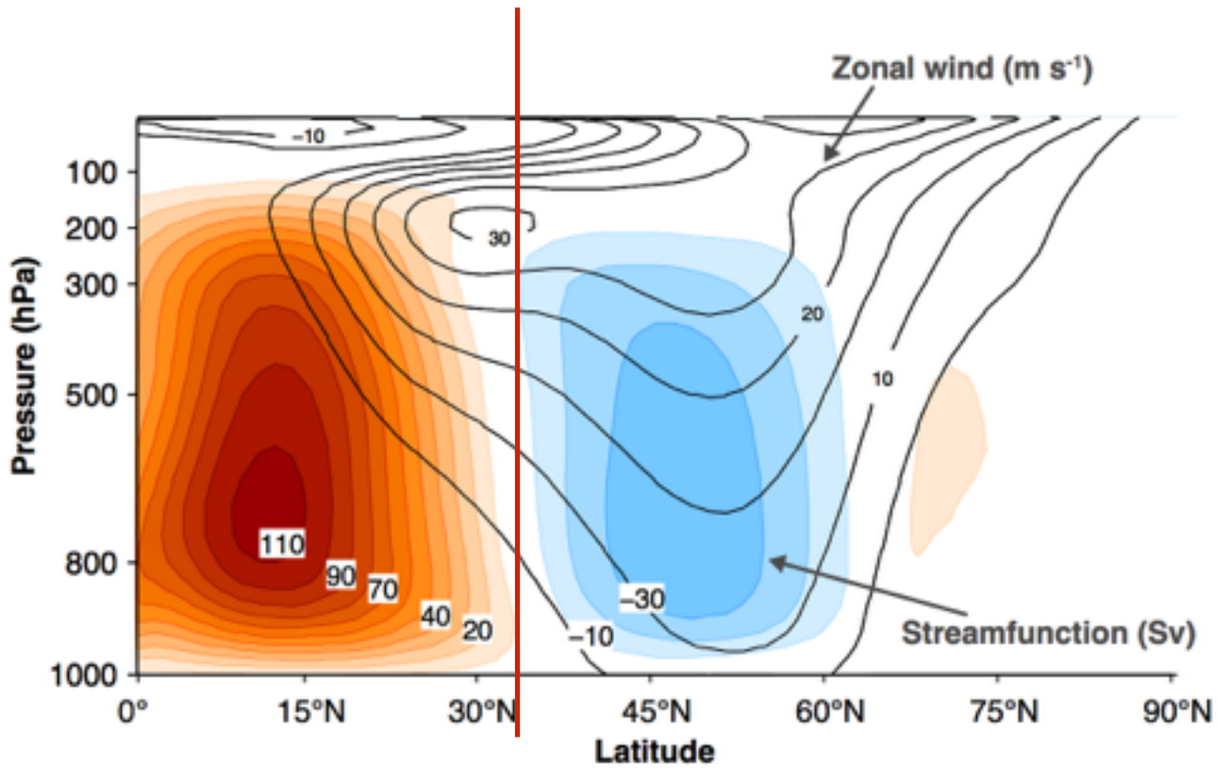
$$\leq 2u_{\mathrm{MAX}}\Omega \sin\phi$$



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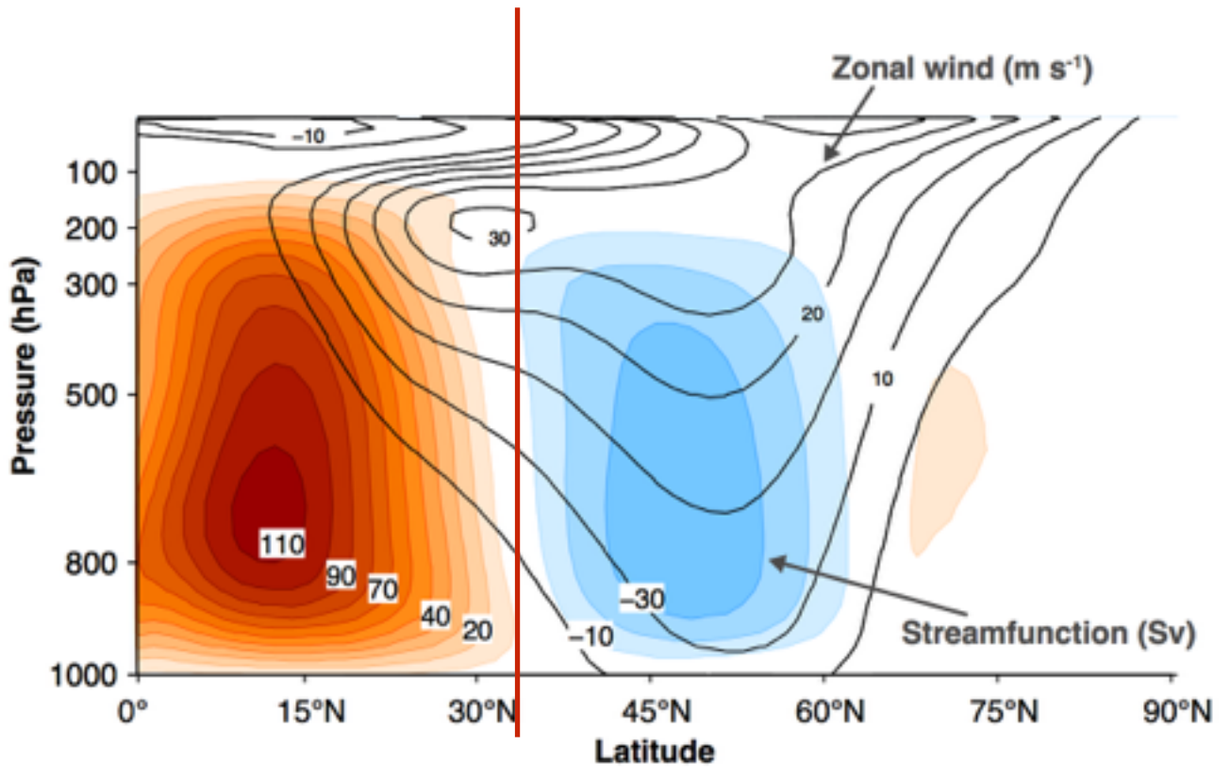

$$-\frac{1}{\rho}\frac{\partial p}{\partial \phi} \leq 2\Omega^2 a^2 \frac{\sin^3\phi}{\cos\phi} \approx 2\Omega^2 a^2 \phi^3$$

using our result from earlier and the small angle approximation

Winds, Hadley circulation, and temperature gradients

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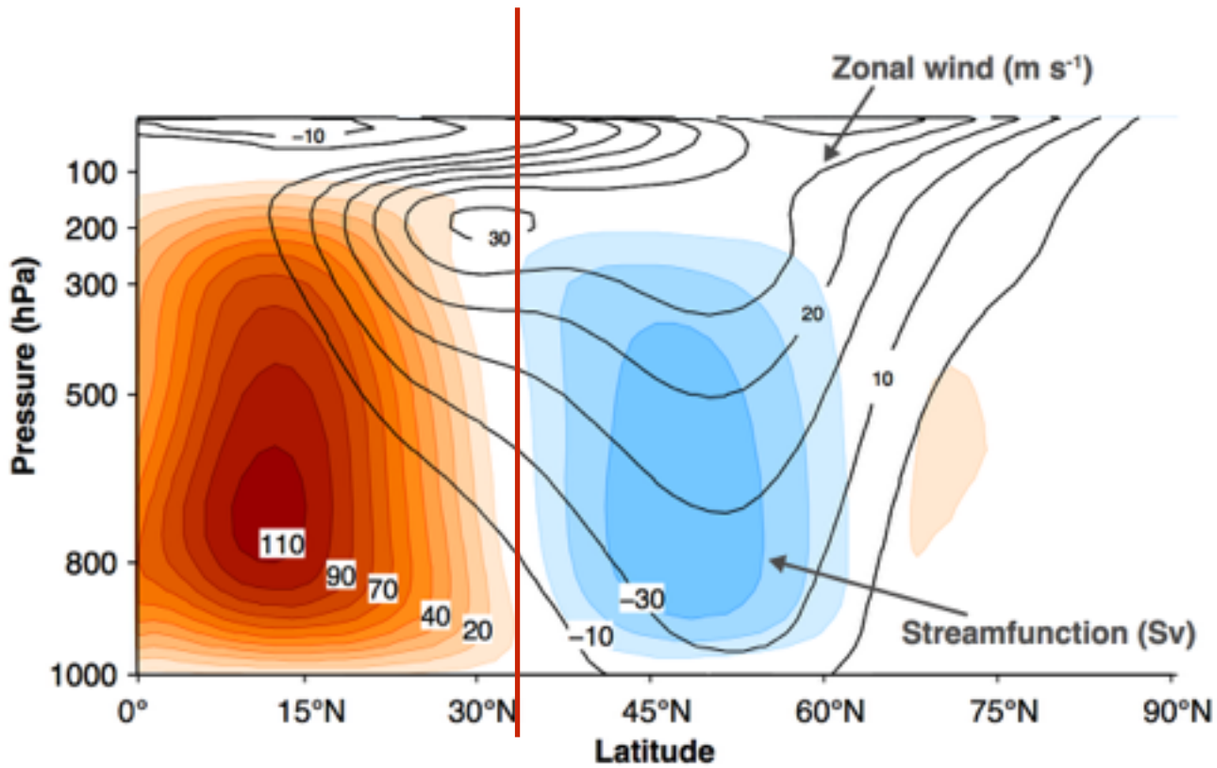


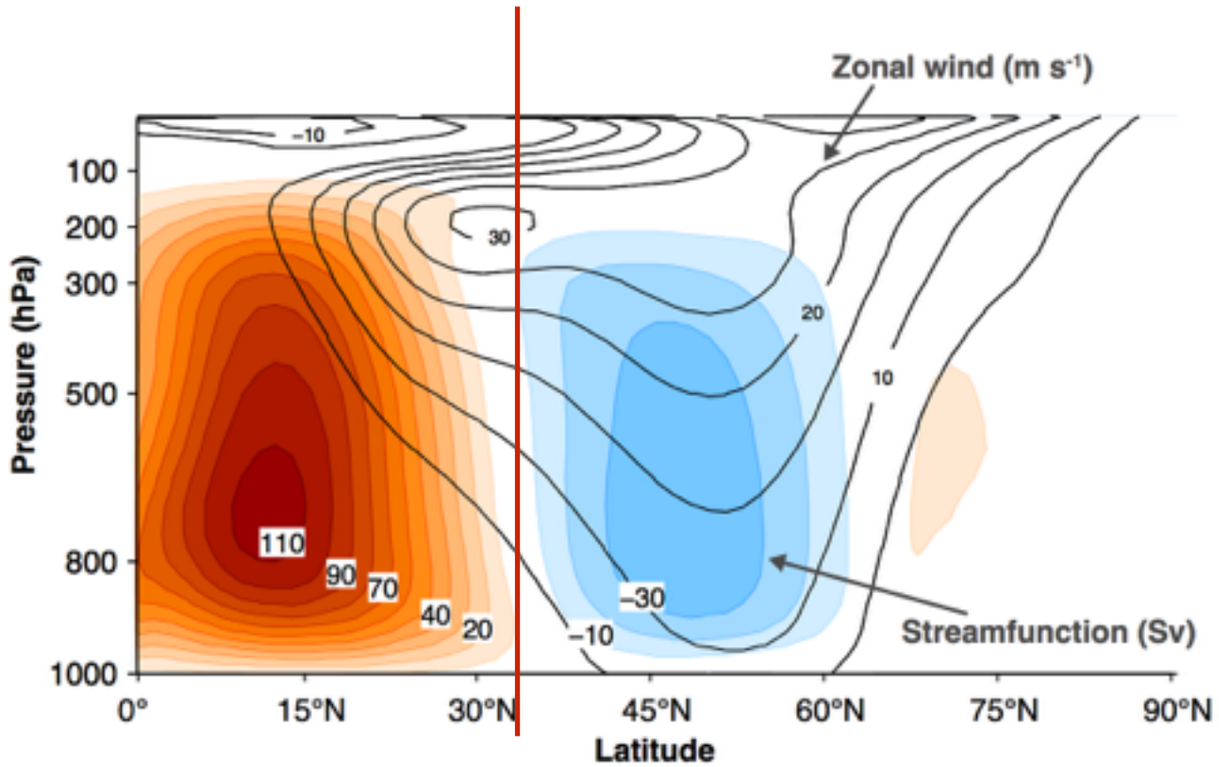
$$\Rightarrow \frac{\partial \langle T \rangle}{\partial \phi} \leq \frac{2\Omega^2 a^2 \phi^3}{R \log p/p_0}$$

expressing in terms of temperature gradients

Winds, Hadley circulation, and temperature gradients

- If *T* gradient is too large, Hadley circulation spins up to reduce gradient and satisfy Hide's theorem
- Can estimate minimum extent of the circulation by solving equation below
- Minimum extent about 25deg (depends on temp gradient)

$$\Rightarrow \frac{\partial \langle T \rangle}{\partial \phi} \leq \frac{2\Omega^2 a^2 \phi^3}{R \log p/p_0}$$



Is Hide's theorem always satisfied?

Hide's theorem not always satisfied: Superrotation

- On other planets (Jupiter, Saturn) and maybe in the Earth's past, Hide's theorem not valid - upgradient AM fluxes and maxima away from the surface

$$M_{\max} > \Omega a^2$$

-> westerly winds on the Equator!

(see video)

Hide's theorem not always satisfied: Superrotation

Caballero & Huber (2010)

- Some evidence to suggest past superstation on Earth
- Models can do it at extremely high CO2