CLIMATE MODELLING

Uncertainty in climate-sensitivity estimates

Arising from: G. C. Hegerl, T. J. Crowley, W. T. Hyde & D. J. Frame *Nature* **440,** 1029–1032 (2006)

Based on reconstructions of past temperatures from proxy data, Hegerl *et al.*[1] estimate a confidence interval for climate sensitivity that suggests a substantially reduced probability of very high climate sensitivity compared with previous empirical estimates. Here I show that the inference procedure used by Hegerl *et al.* neglects uncertainties in temperature reconstructions and in the estimated climate sensitivity and can even be used to infer that the climate sensitivity is zero with vanishing uncertainty. Similar procedures based on temperature reconstructions from proxy data generally underestimate uncertainties in climate sensitivity.

Hegerl *et al.*[1] relate a given univariate time series, θ_i, composited from climate proxies, to a time series, T_i, of mean temperatures by using an errors-in-variables model

$$T_i - \langle T\rangle = \beta(\theta_i - \langle\theta\rangle + \eta_i) + \varepsilon_i \qquad (1)$$

with regression coefficient β and uncorrelated errors η_i and ε_i with zero mean and variances σ_η^2 and σ_ε^2. The subscript i indexes time, and $\langle\cdot\rangle$ denotes a temporal mean. Estimating the mean values $\langle T\rangle$ and $\langle\theta\rangle$ by sample means $\bar{T}$ and $\bar{\theta}$, Hegerl *et al.* obtain an estimate $\hat{\beta}$ of the regression coefficient β from a calibration period in which proxy data, θ_i, and instrumental temperature data, T_i, overlap. With model (1) and the estimated parameters, they infer expected values of past temperature anomalies ("reconstructed temperature anomalies") as $\hat{T}_i'(\hat{\beta}) = \hat{\beta}(\theta_i - \bar{\theta})$, given proxies θ_i in a reconstruction period. They then simulate temperature anomalies, $\hat{T}_i'$ (EBM), with an energy-balance model (EBM) for a range of forcing and model parameters including the climate sensitivity and determine the likelihoods of the temperature-anomaly time series $\hat{T}_i'(\hat{\beta})$ given EBM parameters from the sum of squares of the residuals $r_i(\hat{\beta}) = \hat{T}_i'(\mathrm{EBM}) - \hat{T}_i'(\hat{\beta})$. From these likelihoods, varying $\hat{\beta}$ within estimated confidence intervals, they obtain the marginal distribution of climate sensitivities that led to the conclusion of a reduced probability of high climate sensitivity.

Because this procedure treats reconstructed temperature anomalies, $\hat{T}_i'(\hat{\beta})$, as if they were known temperature anomalies, taking into account uncertainty only in $\hat{\beta}$, it neglects uncertainties in reconstructed temperature anomalies that contribute to uncertainties in the estimated climate sensitivity. The procedure does not take into account the uncertainties in reconstructed temperature anomalies $\hat{T}_i'(\hat{\beta})$ that arise because the mean values $\bar{T}$ and $\bar{\theta}$ are estimated rather than known and the uncertainties reflected in the error terms ε_i and η_i in model (1). (As a result, the estimated margins of error of reconstructed temperature anomalies vanish if the temperature anomalies vanish; see Table S1 of Hegerl *et al.*[1].)

What should enter the calculation of the likelihoods of the temperature-anomaly time series $\hat{T}_i'(\hat{\beta})$ is the estimated variance of the residuals $r_i(\hat{\beta})$, not just the sample variance proportional to their sum of squares, $\Sigma_i r_i^2$. In addition to the contribution taken into account by Hegerl *et al.*[1] — the contribution associated with the variance var($\hat{\beta}$) of the regression-coefficient estimate — a variance associated with the sample means and the variance $\beta^2\sigma_\eta^2 + \sigma_\varepsilon^2$ of the error terms, contribute to the residual variance. The error variance $\beta^2\sigma_\eta^2 + \sigma_\varepsilon^2$ would contribute to the residual variance even if the parameters β, $\langle\theta\rangle$ and $\langle T\rangle$ were known, which demonstrates that the unaccounted variance contribution is generally non-zero, irrespective of how parameters are estimated. Adding estimates of the unaccounted variances to the sample variance of the residuals does not affect the combination of EBM parameters that minimize the residual variance, but it does affect uncertainty estimates by increasing the minimum residual variance. It is inconsistent to estimate temperature variances as sample variances from the reconstructed anomalies $\hat{T}_i'(\hat{\beta})$ as if they were known anomalies while modelling temperatures according to model (1) (ref. 2). If an inference procedure for instrumental temperatures is used for reconstructed temperatures, the additional variance contributions must be taken into account to avoid underestimation of variances and uncertainties.

As an example of how the procedure used by Hegerl *et al.* leads to underestimation of uncertainties in climate sensitivity, consider a hypothetical proxy time series that is constant and equal to the sample mean $\theta_i = \bar{\theta}$ in the reconstruction period and is otherwise arbitrary. The reconstructed temperature anomalies, $\hat{T}_i'(\hat{\beta})$, are zero for any $\hat{\beta}$. It follows that the sample variance ($\propto \Sigma_i r_i^2$) of the residuals $r_i(\hat{\beta})$ between the reconstructed and simulated temperature anomalies is minimized and is equal to zero for zero climate sensitivity and any value of the forcing parameters (provided that the EBM simulation yields vanishing temperature anomalies for zero climate sensitivity, which can always be achieved with the normalizations of Hegerl *et al.*). For non-zero climate sensitivity, which leads to non-zero residuals, the procedure of Hegerl *et al.* gives a vanishing likelihood of the corresponding time series of reconstructed temperature anomalies, $\hat{T}_i'(\hat{\beta})$. The result for the reconstruction period would be an estimate of zero climate sensitivity with probability one — that is, without any uncertainty. If, additionally, instrumental temperature data in the calibration period are taken into account to estimate climate sensitivity, their relative influence can be made arbitrarily small by extending the hypothetical proxy time series into the distant past, leaving the result of zero climate sensitivity with vanishing uncertainty unchanged.

Even if the inferential errors are corrected, similar procedures based on temperature reconstructions from proxy data generally underestimate uncertainties in reconstructed temperatures and hence in climate sensitivity. Climate proxies are often selected on the basis of their correlations with instrumental temperature data, as in the reconstruction[3] underlying the analysis of Hegerl *et al.*[1]. Using such proxies in regression models to reconstruct past temperatures leads to selection bias[4], resulting in an overestimation of the correlation between proxies and temperatures and an underestimation of uncertainties. There are also structural uncertainties: in the structure of the regression model used to reconstruct temperatures (for example, error terms may be correlated if there are non-climatic, low-frequency variations of proxies) and in the structure of the EBM. Hegerl *et al.* acknowledge such structural uncertainties but do not scrutinize them quantitatively. The structural uncertainties may be large compared with the parametric uncertainties taken into account in the inference procedure[5].

For these reasons, uncertainties in temperature reconstructions and climate sensitivity are greater than those given by Hegerl *et al.*[1].

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Despite Schneider's claim[1], the method we use to estimate equilibrium climate sensitivity from multiple proxy-based reconstructions of the temperature in the Northern Hemisphere[2] does account for uncertainty in reconstructions, including that associated with non-temperature and sampling error in the reconstruction. We arrive at a tighter constraint on climate sensitivity not by neglecting uncertainties, but by combining our wide-tailed proxy-based estimate with an independent estimate of climate sensitivity based on twentieth-century warming.

Schneider maintains that incomplete treatment of uncertainties in proxy reconstructions in our estimate yields an overly narrow estimate of equilibrium climate sensitivity[1]. This impression may have been caused by an inconsistency between the reconstruction erroneously attached to the Supplementary Information of ref. 2, which contained only uncertainty in the amplitude of the reconstruction, and its caption, which referred to both amplitude and sampling uncertainty (this error has now been corrected).

We account for uncertainty in temperature reconstructions as fully as possible. Our estimate of climate sensitivity is based on the sample variance of the residual T_i (using Schneider's terminology) between the proxy reconstruction and energy-balance model (EBM) simulations in response to external forcing. We estimate the probability that the difference between the minimum mean-squared residual and that the residual for any other parameter combination is due to noise by using the statistics $\Sigma_i r_i(\text{parameter})^2 - \Sigma_i r_{i,\min} / \bar{v}ar(r_{i,\min})$ (refs 2, 3).

The residual variance is a sum of the sampled variances of internal climate variability, of non-temperature and spatial sampling noise of the reconstruction η_i, and of model error, which varies with model parameters. Therefore η_i, as sampled in each reconstruction (over hundreds of years), is part of the total residual variance and is accounted for in our estimate. Reconstructions with larger η_i yield larger residual variances, resulting in wider probability density functions for climate sensitivity. Our overall estimate of climate sensitivity from the last millennium is based on several reconstructions, which should have largely independent realizations of η_i, thus reducing dependence of our estimate on a particular realization of η_i.

Furthermore, the hypothetical example given by Schneider[1] assumes a reconstruction that has zero variance over the reconstruction period, while varying randomly over the period of overlap with instrumental data. This example would yield numerical degeneracy in the statistic given here, whereas related examples with small preindustrial variance and poor correlations over the calibration period would yield very wide estimates of climate sensitivity owing to large uncertainty in β. Schneider's example violates the assumption that the relationship between proxy data and target of reconstruction can be estimated from the calibration period. It bears no resemblance to reconstructions used here, where the correspondence to (independently reconstructed) forcing is a strong indication that the reconstructions have skill over the pre-instrumental period[4].

We stress that our overall result of a tighter constraint on climate sensitivity does not arise from palaeoclimatic reconstructions alone, which yield a very wide-tailed probability density function (see Fig. 3a of ref. 2). The tighter constraint arises from combining that estimate with an independent estimate of climate sensitivity based on climate change in the late twentieth century. As our sensitivity probability density function also broadly agrees with other, at least partly independent, evidence that we have not used[5,6], we think that our estimate is conservative and valid, despite remaining uncertainties.

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