Vorticity mixing

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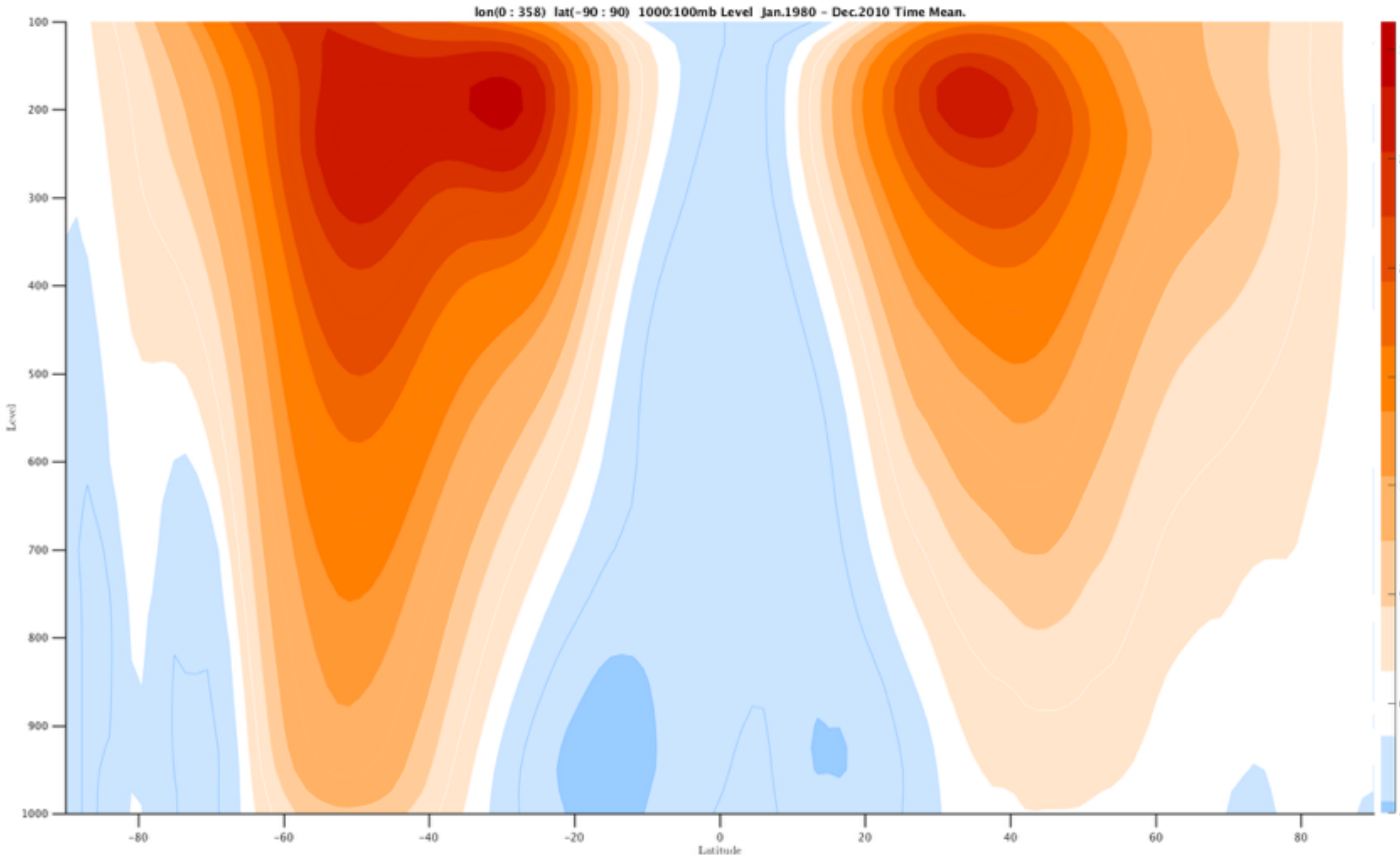
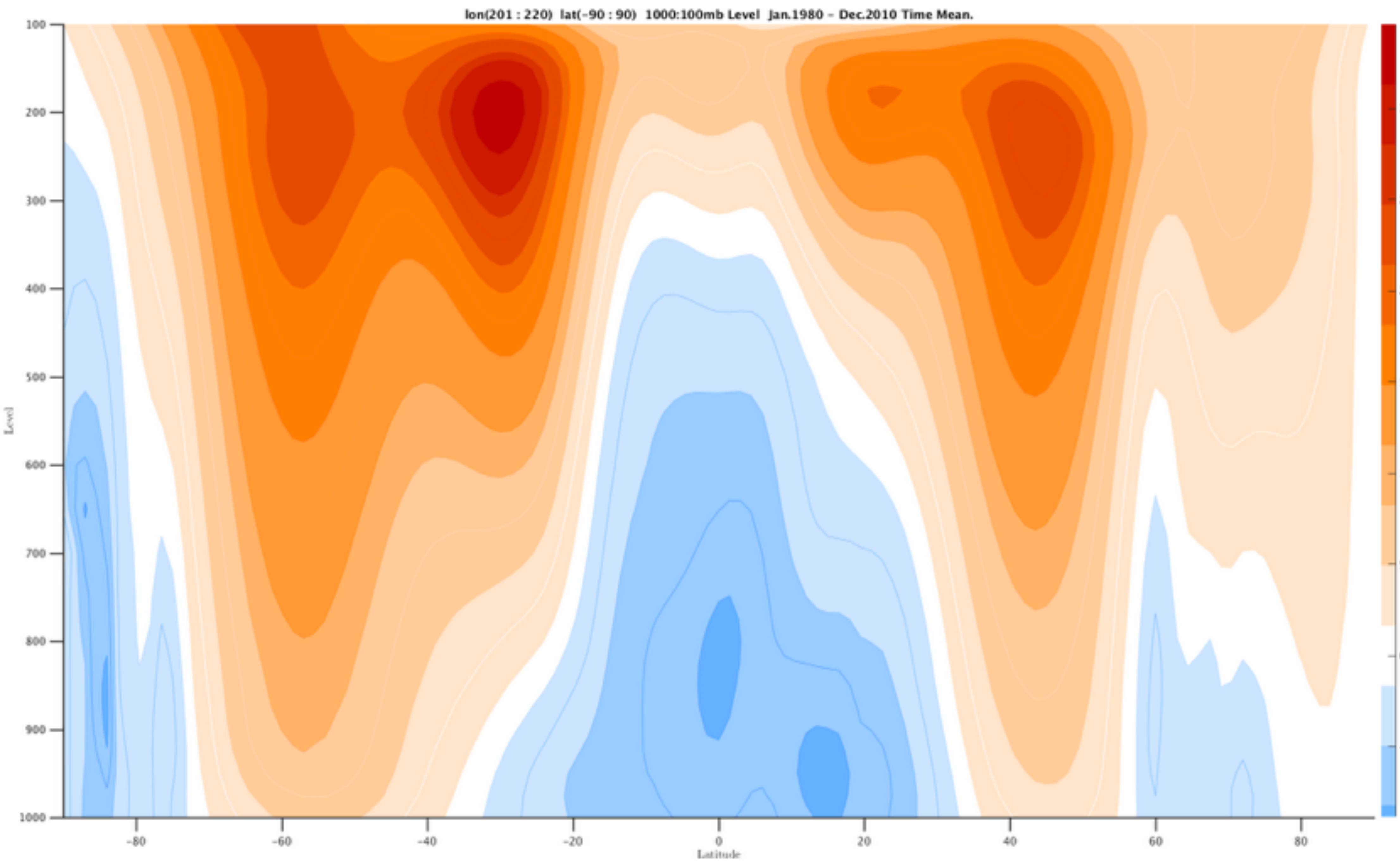
General Circulation ETH

Annual and zonal mean



Annual mean mid Pacific



$$M = \left(u + \Omega r_\perp\right) r_\perp \qquad r_\perp = a\cos(\phi)$$

$$\overline{vM} = \overbrace{\overline{v}\,\overline{M}}^{\text{Mean momentum transport}} + \overbrace{\overline{v'M'}}^{\text{Eddy momentum trasnsport}}$$

Idealized model

- Barotropic: dynamical variables do not depend on z
- Thin shell approximation: *r~a*
- Zonally symmetric boundary conditions
- Incompressible flow

Vorticity

$$\omega = \nabla \times \mathbf{u} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

Vorticity component perpendicular to surface:

$$\eta = \frac{\partial v}{\partial x} - \frac{\partial (U + u)}{\partial y} \qquad U = \Omega a \cos(\phi)$$

$$\eta = \frac{1}{a\cos(\phi)} \frac{\partial v}{\partial \lambda} - \frac{1}{a\cos(\phi)} \frac{\partial}{\partial \phi} \left((U + u)\cos(\phi) \right)$$

$$\eta = \frac{1}{a\cos(\phi)} \frac{\partial v}{\partial \lambda} - \frac{1}{a\cos(\phi)} \frac{\partial}{\partial \phi} \left(u\cos(\phi) \right) + 2\Omega \sin(\phi)$$

$$\underbrace{\eta}_{\text{Absolute vorticity}} = \underbrace{\xi}_{\text{Relative Vorticity}} + \underbrace{f}_{\text{Planetary Vorticity}}$$

Circulation

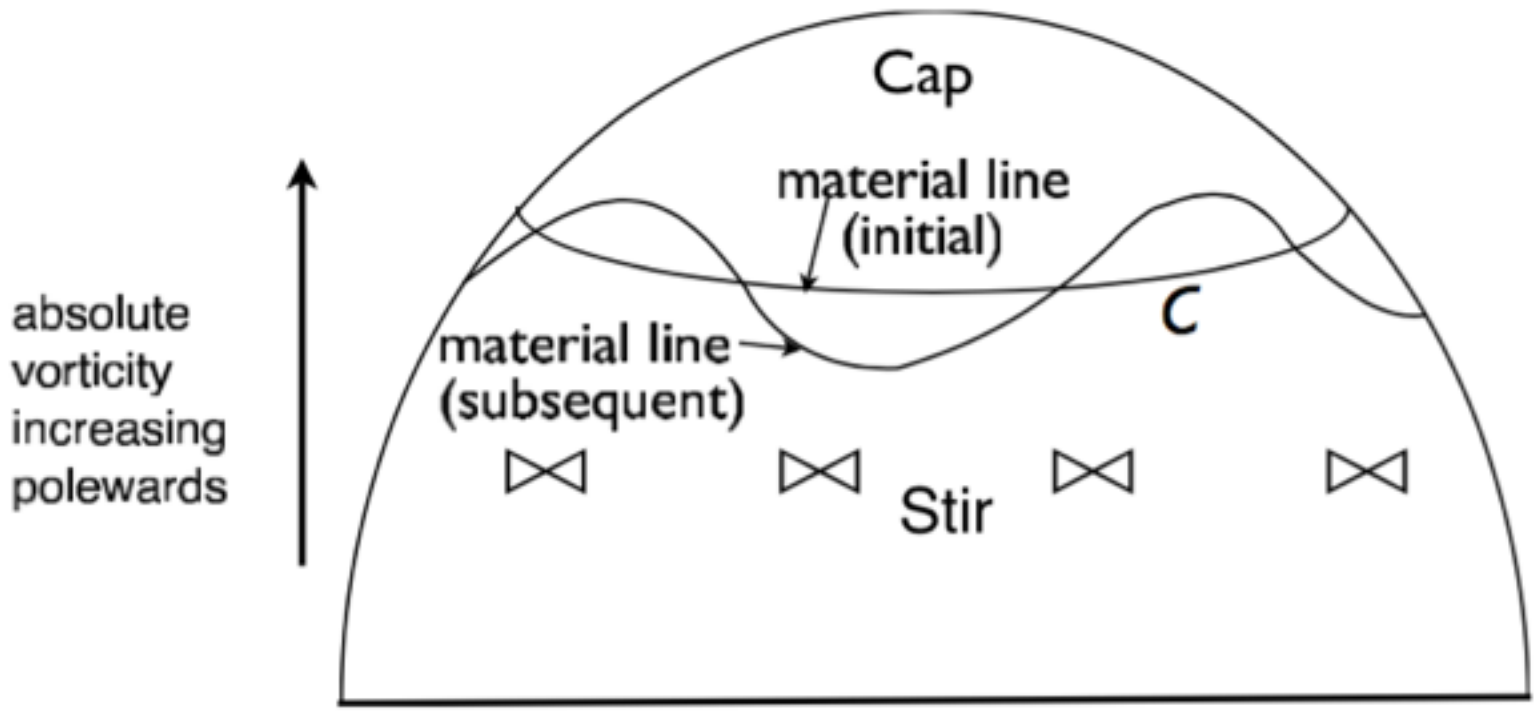
$$C = \oint_l \mathbf{u} \cdot d\mathbf{l} = Circulation$$

Kelvin's theorem: the circulation along a material line is conserved

$$C = \underbrace{\oint_l \mathbf{u} \cdot d\mathbf{l} = \int_A (\nabla \times \mathbf{u}) \cdot d\mathbf{A}}_{\text{Stoke's Theorem}} = \int_A \omega \cdot d\mathbf{A}$$

Locally: $\frac{D}{Dt}(\omega \cdot d\mathbf{A}) = 0$

In our barotropic model: $\frac{D\omega}{Dt} = 0$



Vallis **Fig. 12.2** The effects of a mid-latitude disturbance on the circulation around the latitude line *C*. If initially the absolute vorticity increases monotonically polewards, then the disturbance will bring fluid with lower absolute vorticity into the cap region. Then, using Stokes theorem, the velocity around the latitude line C will become more westwards.

$$C = \oint \left(u + 2\Omega a \cos(\phi) \right) dl$$

- To conserve circulation, u must decrease in the region outside the stirring
- Hence, to conserve net global momentum, u in the stirring region must increase