ESE 133:

Homework 1 (due April 25):

1. **Momentum balance**

   (a) Write down the zonal and meridional momentum balance equations and average them zonally (along latitude circles), decomposing quadratic terms into mean and eddy components.

   (b) What are characteristic scales of the individual terms for large-scale flows? Define a horizontal length scale $L = O(1000\,\mathrm{km})$, a fluctuating velocity scale $U = O(10\,\mathrm{m\,s^{-1}})$, etc. What is the ratio of the advection of momentum by the mean flow and the Coriolis force? What is this ratio called?

   (c) Why are zonal-mean zonal velocities on rapidly rotating planets so much larger than zonal-mean meridional velocities?

   (d) Under what conditions can zonal-mean meridional flow arise?

2. **Angular momentum balance**

   (a) Explain (simply and in words) why the angular momentum component about the spin axis can be written as $M = \Omega r_\perp^2 + u r_\perp = M_\Omega + M_u$, where $r_\perp = r\cos\phi$ (radius r and latitude ϕ).

   (b) Write down the balance equation for M and average it zonally. (You do not need to derive it.) Show how to obtain the zonal momentum equation from the angular momentum equation.

   (c) Show that the advection of the planetary component of the angular momentum, $\mathbf{u} \cdot \boldsymbol{\nabla} M_\Omega$, represents a Coriolis torque.

   (d) From an angular momentum perspective, what is required for a zonal-mean meridional flow to arise? Why does the planetary rotation inhibit zonal-mean meridional flow but not zonal flow?

   (e) By integrating the angular momentum balance along surfaces of constant M_Ω, show that in a statistically steady state there can be no net torque on an M_Ω surface, exerted either by turbulent angular momentum transfer (Reynolds stress) or drag forces.

   (f) Explain briefly how this integral constraint from the angular momentum balance constrains zonal winds.

3. **Extent of Hadley circulations.** Consider nearly inviscid axisymmetric flow in an atmosphere with radiative heating and cooling represented by Newtonian relaxation

$$\partial_t T + \cdots = -\frac{T - T_e}{\tau} \tag{1}$$

toward a radiative-convective equilibrium state T_e with constant relaxation time τ. Give estimates of the minimum latitude ϕ up to which a Hadley circulation must extend if, in the vertical (log-pressure) average $\langle \cdot \rangle$, $\langle T_e \rangle = T_0 - \Delta_h \phi^r$ with $r = 1$, 3, and 4.

4. **Bounds on wind in free troposphere.** Consider a Hadley circulation with ascending branch localized at a latitude ϕ_0 with $|\phi_0| \geq 0$.

    (a) If the flow in the free troposphere is nearly inviscid and the zonal velocity at the boundary is weak, what is the minimum and maximum zonal velocity that can be attained in the free troposphere?

    (b) What are bounds on the vertically averaged temperature gradient?