ESE 133:

Homework 2 (due May 9):

Barotropic Rossby waves. Consider barotropic (two-dimensional) flow on a β-plane with a mean zonal flow $\bar{u}$ that may vary slowly with latitude.

1. Rossby waves and momentum fluxes.

   (a) Show that any purely zonal flow $\bar{u}(y)$ is a solution of the equations of motion.

   (b) Explain in physical terms why Rossby waves always propagate westward relative to the mean flow. (You may want to sketch a Rossby wave and explain how it propagates.) What is the zonal phase speed relative to the mean flow of a Rossby wave with a typical scale of 1500 km, located at 30° latitude?

   (c) Explain why the meridional group velocity of Rossby waves implies that generation of Rossby waves at some latitude y_0 implies convergence of momentum at that latitude. Sketch the associated streamlines of the Rossby waves.

   (d) The most energetic Rossby waves are generated by baroclinic instability in midlatitudes of Earth's atmosphere, where mean zonal flows are strong. They have westerly phase velocities (relative to the solid Earth). As they propagate equatorward, they propagate toward a region of mean easterly winds. Explain why they cannot propagate into that region, and describe qualitatively what happens as they approach it.

   (e) The results of (a)–(d) together imply that Rossby waves generated in midlatitudes of Earth's atmosphere lead to momentum flux divergence in a region of weak westerly or easterly zonal flow, and convergence in a region of stronger westerly zonal flow. In what sense is this momentum flux "upgradient," that is, it is "unmixing" momentum?

   (f) Why is it not as surprising as it might seem at first glance that a turbulent flow leads to "unmixing" of momentum? That is, what kinds of tracers does turbulence tend to homogenize, and why is momentum not one of them?

2. Group velocity of stationary waves and β plumes.

(a) Compute the zonal and meridional group velocity of Rossby waves that are nearly stationary ($\omega \to 0$) and zonally elongated ($k \to 0$).

(b) Why does this group velocity imply that zonally elongated stationary waves extend westward from their source region if their meridional scale is sufficiently large? What does "sufficiently large" mean in this case? [These westward elongated stationary waves are called β plumes, because β is responsible for their existence.]

(c) Consider a stationary wave triggered, e.g., by monsoonal outflow at $25°$N. Assume that the mean zonal flow $\bar{u}$ vanishes at this latitude, and that the meridional scale of the wave that is excited is 3,000 km. What is the zonal group velocity of the wave? If this β plume is thermally damped, and the thermal relaxation time in the atmosphere is 15 days, about how far westward from its source region will the plume extend?