ESE 101:

Homework 5 (due November 16):

1. **Greenhouse effect.** Consider our simple model of the greenhouse effect from class. Assume an optically thin stratosphere that is isothermal at the skin temperature $2^{-1/4}T_e$, where T_e is Earth's effective temperature. Assume Earth's troposphere has a constant temperature lapse $\Gamma = 6.5\ \mathrm{K\ km^{-1}}$, and the current surface temperature is 290 K. Our simple atmosphere has only well-mixed infrared absorbers. We would like to know how it responds to changes in the concentration of these absorbers.

   (a) Sketch the atmospheric temperature profile with the tropopause height H_t and the emission height H_e, where the temperature $T = T_e$. Why is there a tropopause?

   (b) Assume the emission height H_e is the height of peak longwave emission (maximum radiative energy flux divergence). How does it depend on the concentration of absorbers?

   (c) How much does the surface temperature change when the concentration of absorbers is doubled and quadrupled? Why does the logarithmic dependence arise?

   (d) How much does the tropopause height change when the concentration of absorbers is doubled and quadrupled?

   (e) The solutions in (c) and (d) are not realistic for Earth's atmosphere. Why not?

2. **Eddies and mean flows**. We can decompose atmospheric flow fields into mean fields and eddies in various ways. Typically, we define a time mean

$$\overline{(\cdot)} = \frac{1}{T}\int_0^T (\cdot)\, dt \tag{1}$$

   and a zonal mean

$$[\cdot] = \frac{1}{2\pi}\int_0^{2\pi} (\cdot)\, d\lambda, \tag{2}$$

   where t is time and λ is longitude. We define transient eddies

$$(\cdot)' = (\cdot) - \overline{(\cdot)} \tag{3}$$

as fluctuations around the time mean and stationary eddies

$$\overline{(\cdot)}^{*} = \overline{(\cdot)} - \left[\overline{(\cdot)}\right] \tag{4}$$

as fluctuations around the time and zonal mean.

(a) Show that with these definitions, we can decompose averages of fluxes such as the meridional energy flux vE like

$$\overline{vE} = \bar{v}\bar{E} + \overline{v'E'} \tag{5}$$

$$[vE] = [v][E] + [v^{*}E^{*}] \tag{6}$$

and

$$\left[\overline{vE}\right] = \left[\bar{v}\bar{E}\right] + \left[\overline{v'E'}\right] \tag{7}$$

$$= [\bar{v}]\left[\bar{E}\right] + \left[\bar{v}^{*}\bar{E}^{*}\right] + \left[\overline{v'E'}\right]. \tag{8}$$

Under what circumstances is this decomposition exact?

(b) Explain in what sense (8) is a decomposition into mean circulations, stationary eddies, and transient eddies.