ESE 101:

Homework 6 (due November 27):

1. **Eddies and mean flows**. We can decompose atmospheric flow fields into mean fields and eddies in various ways. Typically, we define a time mean

$$\overline{(\cdot)} = \frac{1}{T}\int_0^T (\cdot)\, dt \tag{1}$$

and a zonal mean

$$[\cdot] = \frac{1}{2\pi}\int_0^{2\pi} (\cdot)\, d\lambda, \tag{2}$$

where t is time and λ is longitude. We define transient eddies

$$(\cdot)' = (\cdot) - \overline{(\cdot)} \tag{3}$$

as fluctuations around the time mean and stationary eddies

$$\overline{(\cdot)}^* = \overline{(\cdot)} - \left[\overline{(\cdot)}\right] \tag{4}$$

as fluctuations around the time and zonal mean.

(a) Show that with these definitions, we can decompose averages of fluxes such as the meridional energy flux vE like

$$\overline{vE} = \bar{v}\bar{E} + \overline{v'E'} \tag{5}$$

$$[vE] = [v][E] + [v^* E^*] \tag{6}$$

and

$$\left[\overline{vE}\right] = \left[\bar{v}\bar{E}\right] + \left[\overline{v'E'}\right] \tag{7}$$

$$= [\bar{v}]\left[\bar{E}\right] + \left[\bar{v}^*\bar{E}^*\right] + \left[\overline{v'E'}\right]. \tag{8}$$

Under what circumstances is this decomposition exact?

(b) Explain in what sense (8) is a decomposition into mean circulations, stationary eddies, and transient eddies.

2. **Semi-gray atmosphere with a window**. Reconsider our semi-gray atmosphere model (transparent to solar radiation, grey for longwave radiation), but now introduce an infrared 'window,' that is, a spectral region where the atmosphere is transparent also to longwave radiation. Assume a fraction β of the total longwave spectrum is transparent, so that in this window, the surface radiates directly to space as a blackbody. Outside this window, the atmosphere remains grey for longwave radiation. If the window extends from wavelength λ_1 to λ_2, we can define the fraction β through

$$\beta = \frac{\int_{\lambda_1}^{\lambda_2} B_\lambda(T)\, dT}{\int_0^\infty B_\lambda(T)\, dT},$$

where B_λ is the Planck function. Assume β is a constant, and assume radiative-convective equilibrium with a fixed tropospheric lapse rate Γ is maintained by atmospheric dynamics. Also assume that the stratosphere is optically thin for longwave radiation.

(a) Explain how the surface temperature and tropopause height change in this model as the longwave opacity (greenhouse gas concentrations) of the atmosphere increases outside the window. (You can use graphics and qualitative explanations, or equations.)

(b) Explain why the stratosphere cools as the surface warms when the longwave opacity increases. A qualitative explanation suffices. *Optional bonus:* Calculate the changes quantitatively.