

1     **Linking Hadley circulation and storm tracks in a conceptual**  
2             **model of the atmospheric energy balance**

3             **CHEIKH MBENGUE<sup>1</sup> \* AND TAPIO SCHNEIDER<sup>2</sup>**

*<sup>1</sup>University of Oxford, Oxford, United Kingdom*

*<sup>2</sup>California Institute of Technology, California, United States of America*

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\* *Corresponding author address:* Cheikh Mbengue, Atmospheric, Oceanic, and Planetary Physics, Sherrington Road, Oxford OX1 3PU, United Kingdom.

E-mail: [c.mbengue@wolfson.oxon.org](mailto:c.mbengue@wolfson.oxon.org)

## ABSTRACT

4  
5 Midlatitude storm tracks shift in response to climate change and natural climate variations  
6 such as El Niño, but the dynamical mechanisms controlling these shifts are not well estab-  
7 lished. This paper develops an energy balance model that shows how shifts of the Hadley  
8 cell terminus and changes of the meridional energy flux out of the Hadley cell can drive shifts  
9 of storm tracks, identified as extrema of the atmospheric meridional eddy energy flux. The  
10 distance between the Hadley cell terminus and the storm tracks is primarily controlled by  
11 the energy flux out of the Hadley cell. Because tropical forcings alone can modify the Hadley  
12 cell terminus, they can also shift extratropical storm tracks, as demonstrated through simu-  
13 lations with an idealized GCM. Additionally, a strengthening of the meridional temperature  
14 gradient at the terminus and hence of the energy flux out of the Hadley cell can reduce  
15 the distance between the Hadley cell terminus and the storm tracks, enabling storm-track  
16 shifts that do not parallel shifts of the Hadley cell terminus. Thus, with the aid of the en-  
17 ergy balance model and supporting GCM simulations, a closed theory of storm-track shifts  
18 emerges.

# 1. Introduction

Midlatitude storm tracks are important components of Earth’s general circulation. They redistribute large amounts of energy, moisture, and angular momentum within the atmosphere, and so determine weather and climate patterns over Earth’s surface. Storm tracks have been characterized in several ways in the literature (Chang et al. 2002). A common method uses eddy fields bandpass-filtered to synoptic time scales and identifies storm tracks as regions of large eddy amplitudes (Blackmon 1976; Blackmon et al. 1977; Hoskins and Valdes 1990). It is also possible to use statistics obtained from tracking individual cyclones and anticyclones (Murray and Simmonds 1991; Hoskins and Hodges 2002). Generally, although not always, such different ways of identifying storm tracks agree broadly, in the identification of both, storm tracks in the present climate and their changes in different climates.

It has been extensively described how the structure, strength, and location of midlatitude storm tracks changes as the climate changes (Geng and Sugi 2003; Yin 2005; Bengtsson et al. 2006; Ulbrich et al. 2008; O’Gorman 2010; Barnes and Polvani 2013; Chang 2013; Simpson et al. 2014). These changes affect precipitation and severe weather patterns across midlatitudes. Thus, it is important to understand the mechanisms controlling the storm-track response to perturbations in climate. Although several theories of storm-track shifts have been suggested (Kushner and Polvani 2004; Yin 2005; Chen and Held 2007; Lorenz and DeWeaver 2007; Chen et al. 2008; Lu et al. 2010; Butler et al. 2010; Kidston et al. 2010; Riviere 2011; Butler et al. 2011; Lorenz 2014), a generally accepted one remains elusive because dynamical actors feed back onto each other or act in compensating ways. A closed theory linking them remains outstanding.

To understand the mechanisms of storm-track shifts, in a previous study we used a dry idealized general circulation model (GCM) to link shifts in midlatitude storm tracks to shifts in near-surface midlatitude temperature gradients (Mbengue and Schneider 2013, 2017; hereafter MS13 and MS17). We observed that the Hadley cell terminus often shifts

46 in tandem with the storm tracks, as also seen by Kang and Polvani (2011) and Ceppi and  
47 Hartmann (2013). It is possible that the two influence each other, but the causal link  
48 between them had remained unclear. Some studies suggest that extratropical dynamics lead  
49 to changes in the extent of the Hadley cell. For example, Chen and Held (2007) posit that  
50 changes in the eastward phase speed of extratropical eddies modify the subtropical wave-  
51 breaking latitude, where eddy angular momentum fluxes diverge, and as a result modify  
52 the Hadley cell terminus, considered to be the latitude where the eddy angular momentum  
53 flux divergence changes sign (Korty and Schneider 2008). Alternatively, the extent of the  
54 Hadley cell may be controlled by measures of baroclinicity, such as the subtropical static  
55 stability and meridional temperature gradients, and may change as these change (Walker and  
56 Schneider 2006; Frierson et al. 2007b; Lu et al. 2007; Korty and Schneider 2008; O’Gorman  
57 2011; Mbengue and Schneider 2013; Levine and Schneider 2015). The same baroclinicity  
58 measures also affect the storm track position, but this view leaves unclear how storm tracks  
59 and the Hadley cell terminus are dynamically linked.

60 The idea that shifts in the Hadley cell terminus could cause shifts in storm tracks was  
61 conjecture until numerical experiments by Schneider (2004) and MS13 showed that storm  
62 tracks shift poleward in response to increases in convective static stability in the deep tropics  
63 alone (within  $\pm 10^\circ$  latitude). The forcing, unable to affect extratropical eddies directly, needs  
64 a tropical mechanism by which it can be translated into a poleward shift of midlatitude storm  
65 tracks. The tandem shifts of the Hadley terminus with the storm tracks make the Hadley  
66 circulation a candidate for communicating the changes in tropical convective static stability  
67 to the extratropics. MS17 linked shifts in midlatitude storm tracks, identified as eddy kinetic  
68 energy maxima, to shifts in maxima of near-surface meridional temperature gradients (or of  
69 baroclinic mean available potential energy more generally). Because meridional eddy energy  
70 fluxes in midlatitudes have a diffusive character (Kushner and Held 1998; Held 1999), maxima  
71 of near-surface meridional temperature gradients approximately coincide with maxima of  
72 meridional eddy energy fluxes, which have also been used to identify storm tracks (e.g.,

73 Schneider and Walker 2006, 2008). Therefore, it remains to link shifts in the Hadley cell  
74 terminus to shifts in near-surface meridional temperature gradients or eddy energy fluxes.  
75 Doing so would yield a closed theory for storm-track shifts, at least in dry atmospheres.

76 In this paper, we develop a zonal-mean energy balance model (EBM, see North et al.  
77 (1981) for a review) that links Hadley cell changes to extratropical temperature gradients  
78 and meridional energy fluxes. EBMs have a long history, and important insights have been  
79 derived from them (Budyko 1969; Sellers 1969; Held and Suarez 1974; North 1975a,b; Lindzen  
80 and Farrell 1981; Flannery 1984; Roe and Lindzen 2001; Frierson et al. 2007a). While  
81 they have primarily focused on energy transport by extratropical eddies, Hadley circulation  
82 dynamics have also been incorporated in some EBMs (e.g., Lindzen and Farrell 1981). What  
83 is new about the EBM used here is that it models energy transport by the Hadley cell and  
84 extratropical eddies explicitly and interactively through diffusive closures, with an enhanced  
85 diffusivity within the Hadley cell to represent its strong energy transport. We link shifts  
86 in the Hadley cell energy transport to shifts in near-surface temperature gradients, energy  
87 fluxes, and, by extension, to storm-track shifts.

88 Because the tropical Hadley cell is controlled by complex and generally not diffusive  
89 dynamics (e.g., Schneider and Walker 2006; Schneider et al. 2010) and extratropical eddy  
90 energy transport has contributions from non-local latent energy transport (Pierrehumbert  
91 2002; Pierrehumbert et al. 2007; O’Gorman and Schneider 2006, 2008), local diffusive param-  
92 eterizations of meridional turbulent energy transport may seem questionable at first glance.  
93 Nonetheless, diffusive parameterizations of meridional energy transport are used widely in  
94 EBMs (e.g., Sellers 1969; Held and Suarez 1974; North 1975a,b; Frierson et al. 2007a). One  
95 reason is their mathematical expedience. Additionally, there is a weak separation of the  
96  $O(1000 \text{ km})$  scales of Lagrangian air parcel displacements and the planetary scales over  
97 which near-surface temperature varies meridionally, providing some justification for diffusive  
98 eddy closures in the extratropics, at least for dry static energy fluxes (e.g., Corrsin 1974;  
99 Held 1999; Schneider et al. 2015). Empirically, diffusive closures have also been found to

100 work well on sufficiently long timescales in the extratropics, even for moist static energy  
101 fluxes (Lorenz 1979; Kushner and Held 1998; Frierson et al. 2007a; Roe et al. 2015). In the  
102 tropics, the primary function of the parameterized energy flux in our model is to strongly  
103 reduce meridional temperature gradients (Charney 1963; Sobel et al. 2001), and the precise  
104 form of the transport turns out to be unimportant for our results.

105 Section 2 gives a brief description of the idealized GCM that motivates and supports our  
106 investigations. Section 3 presents the diffusive EBM and the Hadley circulation parametriza-  
107 tion. This is followed by an analysis of solutions of the EBM and idealized dry GCM simu-  
108 lations in Section 3. Section 4 synthesizes and discusses the implications of this work.

## 109 **2. Idealized GCM**

110 The GCM uses the dry dynamical core of the GFDL’s Flexible Modeling System to solve  
111 the primitive equations on a sphere using a spectral transform method at T85 horizontal  
112 resolution, with 30 unevenly-spaced sigma levels. The GCM is the same one used in MS13  
113 and MS17; it is described in detail in Schneider (2004) and Schneider and Walker (2006).  
114 The model is forced using Newtonian relaxation towards a statically unstable radiative-  
115 equilibrium profile on a timescale of 7 days near the surface and 50 days away from the  
116 surface. Standard Earth values are used for physical parameters within the model. The  
117 model has no continents, and no explicit representation of moisture or latent heat release in  
118 phase changes of water.

119 In the GCM, convection is parameterized using a quasi-equilibrium scheme that relaxes  
120 temperatures in an atmospheric column towards a prescribed lapse rate  $\gamma\Gamma_d$  whenever the  
121 column is convectively less stable. Here,  $0 < \gamma \leq 1$  is a rescaling parameter and  $\Gamma_d$  is the dry  
122 adiabatic lapse rate. Increasing  $\gamma$  increases the convective lapse rate and reduces the static  
123 stability of the temperature profile to which the convection schemes relaxes atmospheric  
124 columns. A rescaling parameter  $\gamma < 1$  mimics some of the effects of latent heat release

125 in moist convection, in that it leads to convective temperature profiles that are statically  
126 more stable than a dry adiabat. In MS13 and MS17, we varied the rescaling parameter  $\gamma$   
127 separately near the equator ( $\gamma_e$ , at latitude  $|\phi| < 10^\circ$ ) and in the rest of the atmosphere  
128 ( $\gamma_x$ ) to investigate the effect of deep-tropical static stability changes on extratropical storm  
129 tracks.

### 130 **3. Diffusive EBM**

131 We construct a zonal-mean EBM using the elements we deem important for understand-  
132 ing the mechanisms of storm-track shifts seen in dry atmospheres. Temperature in the EBM  
133 represents near-surface temperatures just above the boundary layer because diffusive eddy-  
134 transport closures have been found to apply best near the surface (Kushner and Held 1998;  
135 Held 1999). The midlatitude storm-track latitude is identified as the latitude of maximum  
136 absolute value of meridional temperature gradients, which was found to be a good indicator  
137 of storm-track latitude in the dry GCM simulations (MS17). With diffusive eddy flux clo-  
138 sures, the latitude of maximal meridional temperature gradients corresponds to the latitude  
139 of maximum meridional eddy energy flux, or approximately (up to cosine factors), the lati-  
140 tude of zero meridional eddy energy flux divergence. The storm-track problem is distilled to  
141 a question of the interactive relationship among near-surface meridional temperature gradi-  
142 ents, Hadley circulation dynamics, and meridional eddy energy transport.

143 Our EBM lacks several processes postulated to be important for storm-track shifts. For  
144 example, there is no explicit representation of eddy phase speed feedbacks, nor are the  
145 effects of changes in extratropical static stability represented. Moreover, the model does  
146 not consider the vertical structure of the atmosphere beyond static stability effects that  
147 are included in the Hadley cell parameterization. The principal advantage of the EBM  
148 is its simplicity. Any storm-track shifts within this EBM stand a better chance of being  
149 mechanistically explainable. Additional processes not considered here (e.g., energy transport

150 by stationary eddies) may be added later, or their effect on storm tracks can be inferred to  
 151 the extent they can be taken as independent of transient eddies.

152 The diffusive EBM equation in spherical coordinates can be written as

$$\partial_t T(\phi) = \frac{\partial_\phi [D(\phi, \phi_h) \cos \phi \partial_\phi] T(\phi)}{R^2 \cos \phi} - \frac{T(\phi) - E(\phi)}{\tau_{\text{rad}}}, \quad (1)$$

153 with polar boundary condition

$$\partial_\phi T(-\pi/2) = \partial_\phi T(\pi/2) = 0. \quad (2)$$

154 Here,  $T$  is temperature,  $\phi$  is latitude,  $D(\phi, \phi_h)$  is the diffusivity, which may be latitude  
 155 dependent and also depends on the Hadley cell terminus at latitude  $\phi_h$ ;  $R$  is the radius  
 156 of the Earth;  $E(\phi)$  is the radiative-equilibrium temperature profile; and  $\tau_{\text{rad}}$  is a radiative  
 157 relaxation timescale. This radiative forcing is similar to that in the GCM in MS13 and  
 158 MS17.

159 The energy balance (1) states that the evolution of temperature (or energy) in a given  
 160 latitude band (left-hand side) is balanced by the energy flux into that latitude band (first  
 161 term on the right) and by the net diabatic heating or cooling in the latitude band (second  
 162 term on the right). Meridional energy fluxes at the poles are zero (2). The forcing and flux  
 163 terms are specified as follows.

164 *a. Radiative forcing*

165 Shortwave heating, long-wave cooling, and sensible and latent surface energy fluxes are  
 166 parametrized using linear relaxation toward a prescribed radiative-equilibrium temperature  
 167 profile  $E(\phi)$  over a fixed radiative timescale  $\tau_{\text{rad}}$ . The radiative-equilibrium profile  $E(\phi)$  is  
 168 the same as in the idealized dry GCM and mimics annual-mean conditions on Earth,

$$E(\phi) = \bar{T}_E + \Delta_H(1/3 - \sin^2 \phi). \quad (3)$$

169 Here,  $\bar{T}_E$  is the global-mean temperature and  $\Delta_H$  is the pole-equator temperature contrast  
 170 in radiative equilibrium.



171 *b. Eddy energy transport*

172 The meridional temperature (energy) flux can be decomposed as

$$[\overline{vT}] = [\overline{v}][\overline{T}] + [\overline{v'T'}] + [\overline{v^* T^*}], \quad (4)$$

173 where the first term on the right-hand side is the meridional energy flux owing to mean  
 174 circulations, the second term is the transient eddy energy flux, and the third is the stationary  
 175 eddy energy flux (see Peixoto and Oort 1992). The overbar represents a temporal mean,  
 176 and primes departures therefrom. The square brackets represent a zonal mean along a  
 177 latitude circle, and asterisks departures therefrom. The GCM we have used in the previous  
 178 studies, which we also refer to throughout this paper, has no means to excite stationary  
 179 zonal asymmetries; hence, we ignore the last term on the right-hand side for now. Thus, a  
 180 parameterization for the meridional energy transport requires finding a relationship for the  
 181 flux  $[\overline{vT}] = [\overline{v}][\overline{T}] + [\overline{v'T'}]$ .

182 Transient eddy energy transport is modeled diffusively. We assume that the diffusive  
 183 fluxes are proportional to the mean meridional temperature gradient, so that the meridional  
 184 flux divergence in spherical coordinates becomes

$$\text{div}[\overline{v'T'}] = -\frac{\partial_\phi [D(\phi, \phi_h) \cos(\phi) \partial_\phi] T}{R^2 \cos(\phi)}. \quad (5)$$

185 In the extratropics ( $|\phi| > \phi_h$ ), we use a constant diffusivity,  $D(\phi, \phi_h) = D_x$ , obtained from the  
 186 empirical mean near-surface eddy diffusivity in the GCM simulations. Since transient eddies  
 187 dominate the energy transport in the extratropics and have a fairly barotropic structure  
 188 (Peixoto and Oort 1992; Schneider and Walker 2008), this is an adequate representation of  
 189 the total transport in the extratropics (Held 1999).

190 Figure 1 shows the near-surface eddy diffusivity  $D = -\{\overline{v'\theta'}\}/\{\partial_y \theta\}$  computed from the  
 191 GCM. Here, the curly bracket means a near-surface vertical average between  $\sigma = 0.8$  and  
 192  $\sigma = 1.0$ . Figure 1a shows the eddy diffusivity in simulations in which the mean radiative-  
 193 equilibrium temperature is varied, with an Earth-like convective lapse rate ( $\gamma_e = 0.7$ ) in

194 the deep tropics and a dry adiabatic convective lapse rate ( $\gamma_x = 1$ ) in the extratropics.  
 195 Figure 1b shows the eddy diffusivity response to changes solely in deep-tropical convective  
 196 stability ( $\gamma_e$ ), while temperatures outside the deep tropics continue to be relaxed toward  
 197 a dry adiabat ( $\gamma_x = 1$ ). In both sets of simulations, the strongest variations in diffusivity  
 198 with latitude occur in the subtropics. In the simulations varying mean temperatures, the  
 199 diffusivity varies with climate outside the Hadley cell terminus. In contrast, in simulations  
 200 varying deep-tropical convective stability, the diffusivity is relatively constant with climate  
 201 outside the Hadley terminus. While approximating the eddy diffusivity  $D_x$  through a con-  
 202 stant in the EBM is not quantitatively accurate, Fig. 1 shows that this may be a useful first  
 203 approximation for purposes of understanding how the storm track position relates to the  
 204 atmospheric energy balance.

205 The variations of diffusivity with latitude and climate in Fig. 1 suggest that a more  
 206 accurate representation would result from using a diffusivity that depends on latitude and  
 207 climate. The maximum near-surface eddy diffusivity in the GCM simulations occurs just  
 208 poleward of the Hadley cell terminus, implying enhanced eddy transport there. There is a  
 209 modest poleward shift in the maximum of the near-surface eddy diffusivity with increasing  
 210 mean radiative-equilibrium temperature, and a significant poleward shift with increasing  
 211 deep-tropical convective stability. The eddy diffusivity’s magnitude also increases as the  
 212 mean radiative-equilibrium temperature or the convective stability increases. These changes  
 213 in diffusivity suggest that eddy length and/or velocity scales have changed (Swanson and  
 214 Pierrehumbert 1997; Held 1999). One may try to capture them by using a diffusive pa-  
 215 rameterization that is a function of the meridional temperature or potential temperature  
 216 gradient,  $\partial_y \bar{\theta}$ . Eddy potential temperature fluxes in dry GCM simulations generally scale  
 217 with a function of static stability and MAPE, which in strongly baroclinic regimes imply  
 218 an approximate  $(\partial_y \bar{\theta})^{3/2}$  dependence of the fluxes (Schneider and Walker 2008). This would  
 219 suggest a diffusivity that depends like  $(\partial_y \bar{\theta})^{1/2}$  on potential temperature gradients. However,  
 220 this dependence is weak, and so we neglect it here. It will turn out that the EBM captures

221 essential features of the storm track shifts seen in the GCM with a constant extratropical  
222 diffusivity.

223 *c. Hadley cell energy transport*

224 In the tropics ( $|\phi| < \phi_h$ ), transport by the mean meridional circulation must also be  
225 considered. To represent it, we use an enhanced diffusivity  $D_t$  at latitudes  $|\phi| < \phi_h$ . This  
226 enhanced diffusivity represents the fact that in the tropics, near the top of the planetary  
227 boundary layer, temperature gradients are rather weak because of efficient redistribution of  
228 energy (Charney 1963; Schneider 1977; Schneider and Lindzen 1977; Lindzen and Farrell  
229 1977; Held and Hou 1980; Sobel et al. 2001).

230 The total diffusivity in the EBM thus takes the form

$$D(\phi, \phi_h) = D_x + (D_t - D_x)\mathcal{S}(\phi, \phi_h), \quad (6)$$

231 where  $\mathcal{S}(\phi, \phi_h)$  is a smoothed top-hat function,

$$\mathcal{S}(\phi, \phi_h) = \frac{1}{2} \left[ 1 - \tanh \left( \pi \frac{\phi - \phi_h}{\phi_h} \right) \tanh \left( \pi \frac{\phi + \phi_h}{\phi_h} \right) \right], \quad (7)$$

232 introduced to smooth the transition between the enhanced tropical diffusivity  $D_t$  and the  
233 weaker extratropical diffusivity  $D_x$ . This parametrization of Hadley cell energy transport  
234 is easy to implement numerically. We use it here because it allows us to study the effects  
235 of Hadley cell expansions on extratropical temperature gradients without complicating the  
236 interpretation of the results. See Mbengue (2015) for a discussion of other Hadley cell  
237 parameterizations we have explored, including one similar to the parameterization in Lindzen  
238 and Farrell (1981).

239 Our overall results are not sensitive to details of how the Hadley cell energy transport is  
240 formulated. However, the transition width of the top-hat function (7) modifies the sensitivity  
241 of the storm tracks to Hadley cell expansions. If the transition between extratropics and  
242 tropics is unphysically abrupt, the tropics and extratropics decouple and thus, Hadley cell

243 expansions push storm tracks poleward only when the Hadley cell terminus is closer to the  
 244 storm tracks.

245 *d. Tropical–extratropical transition*

246 It remains to link the Hadley cell terminus  $\phi_h$  to other quantities in the EBM. The latitude  
 247 at which the eddy momentum flux divergence (meridional wave activity flux) changes sign  
 248 corresponds to the latitude at which vertical wave activity fluxes become deep enough to  
 249 reach the upper troposphere (Korty and Schneider 2008; Ait-Chaalal and Schneider 2015).  
 250 So an approximation of the Hadley cell terminus can be obtained as the latitude at which the  
 251 meridional energy fluxes (vertical wave activity fluxes) first reach the tropopause. Building  
 252 on Held (1978), Schneider and Walker (2006) showed that this is the lowest latitude at which  
 253 the supercriticality

$$S_c(\phi) = -\frac{f}{\beta} \frac{\partial_y \bar{\theta}_s}{\Delta_v} = -\tan(\phi) \frac{\partial_\phi \bar{\theta}_s}{\Delta_v}, \quad (8)$$

254 a measure of the depth of baroclinic eddy energy fluxes, first reaches an  $O(1)$  value. Here,  
 255  $f$  is the planetary vorticity,  $\beta$  is the planetary vorticity gradient,  $y = R\phi$  is the meridional  
 256 distance coordinate,  $\theta$  is potential temperature, and  $\Delta_v = -2 \overline{\partial_p \theta_s} (\bar{p}_s - \bar{p}_t)$  is a bulk static  
 257 stability measure (Schneider and Walker 2006). In a wide range of dry and moist idealized  
 258 GCM simulations, the supercriticality  $S_c$  or moist generalizations thereof indeed take an  
 259 approximately constant  $O(1)$  value at the Hadley cell terminus (Korty and Schneider 2008;  
 260 O’Gorman 2011; Levine and Schneider 2015).

261 Thus, we take  $S_c = S_{c,h}$ , where  $S_{c,h}$  is an  $O(1)$  constant, as the criterion that determines  
 262 where the Hadley cell terminates in our EBM. Because

$$\phi_h = \tan^{-1} \left( -S_{c,h} \frac{\Delta_v}{\partial_\phi \bar{T}} \right), \quad (9)$$

263 this implies that the Hadley cell widens when the bulk stability  $\Delta_v$  at the Hadley cell terminus  
 264 increases, and it narrows when the meridional temperature gradient  $\partial_y \bar{T}$  strengthens. Note  
 265 that we have replaced the mean potential temperature  $\bar{\theta}$  by the mean temperature  $\bar{T}$  because

266 the EBM is written in terms of temperature. This simplification neglects variations in  
 267 pressure and so evaluates the Hadley cell extent on an isobaric surface. Since  $\theta = (p_0/p)^\kappa T$ ,  
 268 the constant  $S_{c,h}$  absorbs a constant  $(p_{ns}/p_0)^\kappa$ , where  $\kappa$  is the adiabatic exponent,  $p_0$  is a  
 269 reference pressure and  $p_{ns}$  is a representative near-surface pressure. Therefore,  $S_{c,h}$  here is  
 270 smaller than if it is evaluated with potential temperature gradients. This simplification is  
 271 justified by how well it predicts the changes in the Hadley terminus in the dry GCM (Fig.  
 272 6 below).

273 We now have a closed set of equations to study storm-track shifts. Solutions of the  
 274 energy balance equation (1) provide the zonal-mean temperature profile and its derivatives,  
 275 as functions of time and latitude. The bulk stability is taken to be an adjustable parameter,  
 276 and the supercriticality is taken to be constant at the Hadley cell terminus.

## 277 4. EBM Results

278 Table 1 lists EBM parameters and their reference values used throughout this study.  
 279 Most parameter values are the same as those used to force the GCM, especially the radiative  
 280 parameters. Table 2 details how the EBM parameters were varied to explore the model's  
 281 sensitivity to them.

### 282 a. Analytical solution

283 An approximate analytical solution of the EBM is possible in a reference frame fixed on  
 284 the Hadley cell terminus  $\phi_h$  if  $D_x \cos(\phi)$  (rather than  $D_x$  itself) is taken to be constant (see  
 285 appendix). The Hadley cell terminus  $\phi_h$  enters the solution as a parameter. In this ap-  
 286 proximate analytical solution, we specify transient eddy energy fluxes  $F$ , or the meridional  
 287 temperature gradient  $\partial_y T = -F/D$ , at the Hadley cell terminus, which serves as a boundary  
 288 condition on the extratropical temperature profile. The analytical solution does not contain  
 289 any Hadley cell parameterization, since iterations would then be necessary to enforce con-

290 tinuous temperatures and temperature gradients at the Hadley cell terminus. Nonetheless,  
 291 much insight is possible even without the explicit Hadley circulation parametrization that is  
 292 contained in the full EBM.

293 For a two-mode spectral approximation of the analytical solution, Fig. 2a shows the  
 294 response of zonal-mean temperatures outside the Hadley cell to changes in the subtropical  
 295 eddy energy fluxes  $F$  at the Hadley cell terminus (left) and to changes in the eddy efficiency  
 296  $\zeta \propto \tau_{\text{rad}} D/R^2$  (right), which measures the eddy mixing efficiency relative to the radiative  
 297 relaxation (Held 1999). Figure 2b shows the corresponding response of the zonal-mean  
 298 meridional temperature gradient profile, while Fig. 2c shows the zonal-mean meridional  
 299 temperature curvature profile. Storm tracks are identified as maxima of the magnitude of  
 300 the meridional temperature gradient, that is, as extrema of the meridional eddy energy flux  
 301 weighted by the cosine of latitude (Fig. 2b). That means they coincide with zeros of the  
 302 meridional energy flux convergence, or zeros of the meridional curvature of the temperature  
 303 profile (Fig. 2c). As the temperature gradient at and thus the transient eddy energy flux  
 304 across the boundary of the Hadley cell strengthen, storm tracks (temperature gradient or  
 305 energy flux extrema) shift equatorward. The equatorward shift occurs because the boundary  
 306 condition of zero meridional temperature gradients at the pole implies that inflection points  
 307 of the temperature profile (extrema of the gradient) shift equatorward as eddy energy fluxes  
 308 across the Hadley cell terminus strengthen.

309 More precisely, the approximate analytical solution shows that to first order, the differ-  
 310 ence between the storm track latitude  $\phi_s$  and the Hadley cell terminus  $\phi_h$  is given by

$$\phi_s - \phi_h \approx \frac{8}{R(1 + \zeta)} E_x \left( \frac{F}{D} \right)^{-1} - \frac{8}{3\pi^2} \frac{4 + \zeta}{1 + \zeta} (\pi/2 - \phi_h), \quad (10)$$

311 where  $E_x$  is an average extratropical radiative-equilibrium temperature. The signs were  
 312 chosen for the northern hemisphere (positive latitudes). This expression shows that storm  
 313 tracks move closer to the Hadley cell terminus if transient eddy energy fluxes  $F$  or temper-  
 314 ature gradients  $\partial_y T = -F/D$  at the Hadley cell terminus strengthen. Increasing the eddy  
 315 efficiency  $\zeta$  also reduces the distance between storm tracks and Hadley cell terminus at fixed

316 temperature gradients at the Hadley cell terminus (Fig. 2, right), and it makes the distance  
 317 less sensitive to variations in the temperature gradient at the terminus. The second term on  
 318 the right-hand side of (10) prevents the distance between Hadley cell terminus and storm  
 319 track from exceeding the distance between the Hadley cell terminus and the pole; thus, it  
 320 depends on the Hadley cell terminus  $\phi_h$ , as does the eddy efficiency  $\zeta$ , where the distance  
 321 between Hadley cell terminus and pole enters in the non-dimensionalization of the diffusivity  
 322 (see appendix). Figure 3 illustrates how the distance  $\phi_s - \phi_h$  varies with eddy energy flux  $F$   
 323 at the Hadley cell terminus for different eddy efficiencies  $\zeta$ , for the complete analytical solu-  
 324 tion of the EBM (2 spectral modes) with constant  $D_x \cos(\phi)$  and for the first-order (10) and  
 325 a second-order (A16) approximations. The Hadley cell terminus  $\phi_h$  is taken to be constant in  
 326 the analytical solutions, although in reality, it also depends on the subtropical temperature  
 327 gradient.

328 These results demonstrate two distinct modes of storm-track shifts:

- 329 i. Storm tracks can shift through changes in the transient eddy energy flux or the temper-  
 330 ature gradient at the Hadley cell terminus. A strengthening of the poleward transient  
 331 eddy energy flux or of the temperature gradient at the Hadley cell terminus generally  
 332 leads to a shift of storm tracks toward the Hadley cell terminus, and a weakening to  
 333 a shift away from the terminus—unless compensated, for example, by changes in eddy  
 334 efficiency or in average extratropical radiative-equilibrium temperature  $E_x$ .
- 335 ii. Storm tracks can shift with the Hadley cell terminus. If, for example, the static stability  
 336 in the subtropics changes, the Hadley cell terminus can shift, leading to a concomitant  
 337 shift of the storm tracks provided changes in the transient eddy energy flux across  
 338 the Hadley cell terminus or in the eddy efficiency do not modulate the Hadley cell–  
 339 storm track distance sufficiently to compensate. This is a mechanism through which  
 340 tropical processes (e.g., changes in tropical convective stability) can influence storm-  
 341 track position. It is worth noting that the average extratropical radiative-equilibrium  
 342 temperature  $E_x$  depends on the Hadley cell terminus  $\phi_h$ . Thus, poleward storm-track

343 shifts driven by Hadley cell expansions can be compensated by equatorward shifts due  
344 to decreases in  $E_x$ .

345 These two modes of storm track shifts arise out of the EBM dynamics and exist independently  
346 of changes, for example, in eddy diffusivities or eddy efficiencies. For example, a change in  
347 the eddy length scale, as proposed previously (Kidston et al. 2010), is not necessary; changes  
348 in transient eddy energy fluxes across the Hadley cell terminus suffice, however they arise.  
349 But our results do not rule out that changes in eddy length scales or other factors may still  
350 be important for storm track shifts.

### 351 *b. Numerical simulations*

352 We solve the complete set of EBM equations numerically with second-order central dif-  
353 ferences in space, with  $1^\circ$  resolution. The simulations are advanced in time using forward  
354 Euler time stepping, until a steady state is reached. The simulations advance as follows: (1)  
355 Given a temperature profile as a function of latitude,  $\phi_h$  is calculated as the first latitude  
356 where the supercriticality (8) exceeds a critical value; (2) the diffusivity is calculated based  
357 on (6); (3) the model is integrated one step forward in time using (1); (4) this sequence is  
358 iterated in the next time step.

### 359 1) SENSITIVITY TO TROPICAL STABILITY VARIATIONS

360 To relate the Hadley cell extent to the convective lapse rate in the tropics, we need to  
361 relate  $\Delta_v$  to the convective rescaling parameter  $\gamma$ . Assuming that hydrostatic equilibrium  
362 holds and that the convective lapse rate  $\gamma\Gamma_d$  prevails throughout the tropics, we deduce that  
363  $\partial_p\theta = -(\theta/T)(\Gamma_d - \gamma\Gamma_d)/(\rho g)$ . Hence, the tropical bulk stability is given by

$$\Delta_v = \frac{2(1 - \gamma)(p_s - p_t)\theta}{\rho c_p T}, \quad (11)$$



364 and we assume the subtropical bulk stability at the Hadley cell terminus to scale with the  
365 tropical bulk stability. In using (11), we assume that  $T \approx \theta$ . Thus, we can analytically relate  
366 variations in  $\gamma$  to variations in the bulk stability  $\Delta_v$  at the Hadley cell terminus and, given  
367 the temperature gradient at the terminus, to the Hadley cell extent via the supercriticality  
368 criterion (9). For fixed meridional temperature gradients and supercriticality at the Hadley  
369 cell terminus, increasing the bulk stability leads to a poleward expansion of the Hadley cell.

370 The response of the Hadley cell terminus and the storm tracks in the EBM to changes in  
371 the convective stability are shown in Fig. 4. The EBM’s response is qualitatively similar to  
372 the response seen in the GCM (Fig. 1b): as the convective stability increases, the midlatitude  
373 storm tracks migrate poleward, generally in tandem with the Hadley cell terminus. This  
374 suggests that the EBM captures important mechanisms controlling storm-track shifts in  
375 response to deep-tropical convective-stability variations. The storm tracks in this case are  
376 essentially being forced poleward by an expanding Hadley circulation. As the Hadley cell  
377 expands, maximum temperature gradients are expelled further poleward—the second mode  
378 of storm track shifts highlighted above.

379 In low-convective stability climates in the EBM, the distance from the storm tracks to  
380 the Hadley cell terminus is too great for the expansion of the Hadley cell to be communicated  
381 through midlatitude eddies (Fig. 4, simulations with large  $\gamma$ ). The shift of the Hadley cell  
382 terminus decouples from the storm track shift. A similar behavior is seen in the GCM at  
383 low convective stability (see MS13 or Fig. 1b). In the range  $\gamma \leq 0.88$ , synoptic eddies begin  
384 to feel the influence of the expanding Hadley cell and migrate poleward with it.

385 Forced on its own, the EBM produces results that are qualitatively similar to the GCM,  
386 supporting the hypothesis that shifts in the Hadley cell terminus can push storm tracks  
387 poleward.

389 For a more direct comparison of EBM with GCM results, we take the subtropical bulk  
 390 stability from zonal-mean statistics in the GCM simulations and then solve the EBM nu-  
 391 merically with the given bulk stability, specifying the diffusivity as in (6), with the tropical  
 392 diffusivity  $D_t$  taken to be  $10^7 \text{ m}^2 \text{ s}^{-1}$  and the extratropical diffusivity  $D_x$  is taken to be half  
 393 of the GCM’s extratropical-mean diffusivity, in order for the EBM’s and GCM’s maximum  
 394 extratropical temperature gradient to match. The extratropical diffusivity  $D_x$  is usually  
 395 around  $2 \times 10^6 \text{ m}^2 \text{ s}^{-1}$ . Additionally, we use the global-mean temperature from the GCM as  
 396 the EBM’s mean radiative-equilibrium temperature  $\bar{T}_E$ . The EBM and GCM are forced in  
 397 the same way and occupy different positions in a model hierarchy, with the EBM elucidating  
 398 causes of the storm track shifts seen in the GCM simulations.

399 Figure 5 compares a statistically steady state climate from the GCM to the corresponding  
 400 EBM result. The zonal-mean temperature profiles agree qualitatively (Fig. 5a). The errors  
 401 in the extratropics are largest near the poles, where variations of the effective diffusivity  
 402 with latitude play a sizable role. There are also substantial errors in the deep tropics: for  
 403 example, the EBM’s temperatures are biased low relative to the GCM’s—although they both  
 404 have the same global-mean temperature. This is because a near-surface thermally-indirect  
 405 circulation, not taken into account in the EBM, reverses temperature gradients near the  
 406 equator in the GCM. In simulations where there is no such near-surface circulation, there is  
 407 a closer correspondence between the EBM and the GCM in the tropics—but the errors at  
 408 the pole remain because the constant-diffusivity parameterization is inadequate there.

409 Although the EBM with its constant diffusivity in the extratropics is too simple to capture  
 410 the rich structure of the GCM’s temperature gradient profile (Fig. 5b), it does capture the  
 411 position of the storm track, and the Hadley cell parameterization seems to work adequately.  
 412 The EBM captures the salient features of the GCM’s mean-temperature and temperature-  
 413 gradient profile. The storm track latitudes in the EBM and GCM roughly coincide if the  
 414 tropical and extratropical diffusivities derived from the GCM are used in the EBM.

415 Figure 6a compares storm-track and Hadley-terminus responses to variations in tropical  
 416 convective stability. Here, the EBM is forced with variables computed from the GCM: the  
 417 mean temperature, average extratropical diffusivity (averaged between the Hadley terminus  
 418 and 60N), and bulk stability are all taken from GCM simulations in which the deep-tropical  
 419 convective stability is varied. The diffusivity within the Hadley cell is set to a constant  
 420  $D_t = 10^7 \text{ m}^2 \text{ s}^{-1}$ . The analytical solution for the storm track latitude given the Hadley cell  
 421 terminus latitude, with parameters as in the full EBM, is also shown in the figure. As the  
 422 tropical convective stability increases, the Hadley cell expands and the storm tracks shift  
 423 poleward, in tandem with the Hadley cell terminus. With a constant tropical diffusivity  
 424  $D_t$ , storm tracks in the EBM sit  $5.5^\circ \pm 0.9^\circ$  poleward of the storm tracks in the GCM.  
 425 This is because, as explained earlier, thermally indirect circulations in the deep tropics in  
 426 the GCM lead to steeper subtropical temperature gradients in the GCM, which are not  
 427 captured in the EBM. Figure 6 also reveals that the supercriticality criterion (9) in the  
 428 EBM captures changes in the Hadley cell extent in the GCM very well. Figure 6b is the  
 429 same as in Fig. 6a but instead uses an average tropical diffusivity  $D_t$  computed for each GCM  
 430 simulation individually (by averaging fluxes and gradients between  $10^\circ\text{N}$  and the Hadley cell  
 431 terminus). The fit between the storm tracks in the EBM and GCM is improved, albeit with  
 432 a slight degradation in the Hadley terminus fit.

433 Figure 7 compares the relationship between the subtropical temperature gradients and  
 434 the differences between storm-track latitude and the Hadley cell terminus, in the EBM,  
 435 GCM, and analytical solution of the EBM. What are plotted are the anomalies around  
 436 the arithmetic mean for each dataset. There is a clear monotonic relationship between  
 437 subtropical temperature gradients and the distance between the storm tracks and the Hadley  
 438 cell terminus. In the EBM using the GCM's tropical diffusivity, storm tracks move away from  
 439 the Hadley cell terminus at  $0.27^\circ\text{--}0.31^\circ$  per Kelvin strengthening of subtropical temperature  
 440 gradients (where temperature gradients are multiplied by Earth's radius  $R$  to express them as  
 441 temperature differences). In the GCM, the dependence of storm track distance to Hadley cell

442 terminus on subtropical temperature gradients is, within the statistical uncertainty, similar  
443 ( $0.19^{\circ}$ – $0.71^{\circ}$  per K), and it is also similar for the analytical solution to the EBM ( $0.23^{\circ}$ – $0.85^{\circ}$   
444 per K). Much of the spread in the GCM results around the regression line comes from noise  
445 in estimating subtropical temperature gradients from the GCM. Overall, however, changes  
446 in the subtropical temperature gradient and in the flux across the Hadley cell terminus  
447 account for only a small fraction of the variance in the storm-track latitude. The shifts in  
448 the storm-track latitude in these simulations are dominated by shifts in the Hadley terminus.

## 449 5. Discussion and conclusions

450 We have developed an EBM with an explicit representation of Hadley cell energy trans-  
451 port to illustrate how the storm track position can be dynamically coupled to Hadley cell  
452 dynamics. A key ingredient of this coupling is that the Hadley cell terminates where a baro-  
453 clinicity measure, which depends on the meridional temperature gradient and a bulk static  
454 stability, first exceeds a critical value. The meridional temperature gradient at the Hadley  
455 cell terminus, in turn, determines the eddy energy export out of the Hadley cell into the  
456 extratropics, and its strength controls the distance between the Hadley cell terminus and  
457 the storm tracks, identified as extrema in the meridional temperature gradient or zeros of  
458 the meridional eddy energy flux divergence.

459 While this EBM is simple conceptually, it exhibits rich behavior, which can help explain  
460 how the storm track position varies as various climatic factors are varied. Two distinct  
461 modes of storm track shifts emerge: (1) Storm tracks move closer to the Hadley cell as the  
462 meridional temperature gradient, or transient eddy energy flux, at the Hadley cell terminus  
463 strengthen. This is a result of the diffusive nature of the EBM, which leads to an inflection  
464 point in the meridional temperature profile closer to the Hadley cell terminus when subtrop-  
465 ical temperature gradients strengthen. (2) Storm tracks shift in tandem with the Hadley cell  
466 terminus, as long as changes in the transient eddy energy export out of the Hadley cell (and

467 parameters such as the diffusivity) stay fixed. This is a result of the dynamical coupling  
468 between the latitude of the Hadley cell terminus and meridional temperature gradients in  
469 the extratropics, whose extrema coincide with the storm tracks.

470 Both modes of storm track shifts can be demonstrated in the EBM, both in analytical  
471 and numerical solutions, and they are also seen in the simulations with an idealized GCM we  
472 published earlier (MS13 and MS17). In particular, most variations in storm track latitude  
473 in the GCM are accounted for by variations in the Hadley cell terminus. These can arise  
474 entirely by tropical processes. For example, an increase in the convective stability only in the  
475 deep tropics leads to a widening of the Hadley cell by increasing the bulk stability throughout  
476 the tropics (MS13), thus leading to a poleward shift of the latitude where the baroclinicity  
477 measure first exceeds its critical value. This poleward expansion of the Hadley cell pushes  
478 extratropical temperature gradients and thus storm tracks poleward, because the distance of  
479 storm tracks to the Hadley cell terminus, for a given diffusivity, depends on the meridional  
480 temperature gradient at the Hadley cell terminus as a boundary condition. Such changes in  
481 the Hadley cell terminus can arise through a variety of processes and account for most of the  
482 shifts in storm track latitude seen in the GCM simulations (mode 2 above). Additionally,  
483 changes in the meridional temperature gradient at the Hadley cell terminus lead to changes  
484 in the distance between storm tracks and the Hadley cell terminus, which account for a  
485 smaller portion of the shifts in storm track latitude seen in the GCM simulations (mode 1  
486 above).

487 Our results at least qualitatively provide a unifying conceptual model for various ways  
488 in which storm tracks can shift that have been described in the literature:

- 489 • Idealized and comprehensive climate models show a robust poleward shift of storm  
490 tracks as the climate warms globally (Yin 2005; Bengtsson et al. 2006; Hu and Fu 2007;  
491 Lu et al. 2007; Seidel et al. 2008; Barnes and Polvani 2013; Mbengue and Schneider  
492 2013; Adam et al. 2014; Levine and Schneider 2015). Generally, the storm tracks shift  
493 in tandem with the Hadley cell terminus (Kang and Polvani 2011; Ceppi and Hartmann

494 2013; Mbengue and Schneider 2013). Our EBM likewise predicts a poleward shift of  
495 storm tracks under global warming through expansions of the Hadley cell primarily  
496 driven by increased tropical static stability (mode 2).

497 • In contrast to the global-warming response, storm tracks migrate equatorward during  
498 the El Niño phase of the Southern Oscillation (Chang et al. 2002; Seager et al. 2003;  
499 Lu et al. 2008; Tandon et al. 2013; Adam et al. 2014). During El Niño, subtropical  
500 meridional temperature gradients strengthen, which can lead to the contraction of the  
501 Hadley cell because the subtropical baroclinicity (supercriticality) increases. Consis-  
502 tent with our EBM, storm tracks shift equatorward, most likely primarily with the  
503 Hadley cell terminus (mode 2). It is an open and interesting question whether an ad-  
504 ditional storm track shift toward the Hadley cell (mode 1) can also be extracted from  
505 observations.

506 • Idealized GCM simulations aimed at elucidating the differences between the global-  
507 warming and El Niño responses of the atmospheric circulation have shown equatorward  
508 storm-track shifts for narrow tropical heating or strengthened subtropical temperature  
509 gradients, and poleward storm-track shifts for broad tropical or midlatitude heating  
510 (Chang 1995; Butler et al. 2010; Tandon et al. 2013; Sun et al. 2013). Such storm-track  
511 shifts are consistent with the modes 1 and 2 mechanisms discussed here. For example,  
512 narrow tropical heating strengthens subtropical temperature gradients and leads to a  
513 contraction of the Hadley cell, implying an equatorward storm-track shift both because  
514 the Hadley cell contracts and possibly also because temperature gradients at the Hadley  
515 cell terminus strengthen. On the other hand, broad tropical heating increases the  
516 subtropical bulk stability, leading to an expansion of the Hadley circulation and a  
517 concomitant poleward storm track shift.

518 • In idealized GCM simulations, storm tracks have been seen to move closer to the Hadley  
519 cell as subtropical temperature gradients strengthen and away from it as subtropical

520 temperature gradients weaken (Brayshaw et al. 2008; Tandon et al. 2013)—like in our  
521 EBM and dry GCM simulations.

- 522 • Idealized and comprehensive GCM simulations have shown that storm tracks shift  
523 poleward as the tropopause height increases (Schneider 2004; Williams 2006; Lorenz  
524 and DeWeaver 2007). The same occurs in our EBM and dry GCM because an increased  
525 tropopause height implies an increased bulk stability in the subtropics and thus an  
526 expansion of the Hadley cell, which pushes storm tracks poleward.
- 527 • The EBM, in concert with our previous GCM studies, is consistent with the finding that  
528 variations in lower-tropospheric meridional temperature gradients account for much of  
529 the spread in storm track positions seen in comprehensive GCM simulations (Harvey  
530 et al. 2015).

531 Our EBM notably does not represent upper-tropospheric eddy momentum fluxes explicitly,  
532 except insofar as that the eddy momentum fluxes are viewed as essential for terminating  
533 the Hadley cell where their divergence changes sign, which occurs where the vertical wave  
534 activity fluxes become deep enough to reach the upper troposphere (Korty and Schneider  
535 2008). It has been postulated that changes in eddy phase speeds that modulate where eddies  
536 break and eddy momentum fluxes change sign can drive changes in the Hadley cell terminus  
537 and in storm track latitude (Chen and Held 2007). While that remains a possibility, from the  
538 perspective of the EBM, the upper-tropospheric eddy momentum fluxes adjust to, rather  
539 than drive, changes in the lower-tropospheric eddy energy fluxes (vertical wave activity  
540 fluxes) and temperature gradients that ultimately determine the storm track position.

541 To what extent the EBM can quantitatively account for storm track shifts by a variety  
542 of disparate mechanisms remains to be investigated. It is possible that variations of the  
543 diffusivity/eddy efficiency with latitude and climate may need to be taken into account  
544 to quantitatively account for some of the shifts seen in observations and simulations. It  
545 is also to be kept in mind that storm tracks are strongly influenced by stationary eddy

546 activity (Kaspi and Schneider 2011, 2013; Simpson et al. 2014; Shaw and Voigt 2015), which  
547 are not explicitly taken into account in our EBM. Additionally, moisture and latent heat  
548 release affect the eddy efficiency and static stability (Flannery 1984; Held 1993; Caballero  
549 and Langen 2005; O’Gorman and Schneider 2008; Schneider and Bordoni 2008; O’Gorman  
550 2011; Cabellero and Hanley 2012)—effects that currently are not considered in our EBM but  
551 may be captured with suitable generalizations of the bulk stability or the energy flux that is  
552 represented by the EBM. The EBM and the two modes of storm-track shifts it exhibits are  
553 a useful starting point to anchor such further investigations.



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558

## APPENDIX

### Analytical EBM Solution

559 Starting with the steady-state version of the EBM equation and assuming  $D \cos \phi$  to be  
560 constant, we obtain a Cartesian ODE,

$$D\partial_{yy}T - A[T - E(y)] = 0, \quad (\text{A1})$$

561 where  $y = R\phi$  and  $A = 1/\tau_{\text{rad}}$ . The boundary conditions are a specified energy flux  $F$  across  
562 the Hadley terminus at distance  $y_h = R\phi_h$  from the equator, and zero flux at the pole:

$$\begin{aligned} \partial_y T(y_h) &= -F/D \\ \partial_y T(R\pi/2) &= 0. \end{aligned} \quad (\text{A2})$$

563 Focusing on the northern hemisphere for notational simplicity, we nondimensionalize the  
564 domain using  $y' = \alpha(y - y_h)$ , where  $\alpha = (y_p - y_h)^{-1}$  with distance to pole  $y_p = R\pi/2$ , so that  
565 the Hadley cell terminus is at  $y' = 0$  and the north pole at  $y' = 1$ . The radiative-equilibrium  
566 temperature profile is transformed accordingly to

$$E(y') = \bar{T}_E + \Delta_H \left\{ 1/3 - \sin^2 \left( [\alpha^{-1}y' + y_h]/R \right) \right\}. \quad (\text{A3})$$

567 The ODE becomes

$$D'\partial_{y'y'}T - A[T - E(y')] = 0, \quad (\text{A4})$$

568 where  $D' = \alpha^2 D$ , with boundary conditions,

$$\begin{aligned}\partial_{y'} T(0) &= -\alpha F/D' \\ \partial_{y'} T(1) &= 0.\end{aligned}\tag{A5}$$

569 The EBM equation subject to the boundary conditions and the assumptions above can  
570 be solved analytically. First, we homogenize boundary conditions by defining a new tem-  
571 perature,  $T = T' + w$ , where  $w = -2\alpha F/(\pi D') \sin(\pi y'/2)$ . This yields the transformed  
572 equation

$$[D' \partial_{y'y'} - A] T' + S' = 0,\tag{A6}$$

573 with homogeneous boundary conditions

$$\begin{aligned}\partial_{y'} T'(0) &= 0, \\ \partial_{y'} T'(1) &= 0,\end{aligned}\tag{A7}$$

574 where  $S' = AE - (A + \pi^2 D'/4) w$ , or

$$S' = AE + \frac{\pi \alpha B F}{2} \sin\left(\frac{\pi y'}{2}\right),\tag{A8}$$

575 with

$$B = \left(\frac{4A}{\pi^2 D'} + 1\right)\tag{A9}$$

576 as a measure of radiative forcing strength relative to eddy diffusion. This boundary value  
577 problem has cosines as eigenfunctions, so we express the solution as a cosine series,  $T' =$   
578  $\sum_{n \geq 0} \tau_n \cos(n\pi y')$ , and expand the forcing analogously:  $S' = \sum_{n \geq 0} s_n \cos(n\pi y')$ . The so-  
579 lution for temperature follows immediately by substitution, if we assume a small angle ap-  
580 proximation for the implicit secant variations of  $D$ ,

$$T(y') = w + \sum_{n \geq 0} \frac{s_n \cos(n\pi y')}{n^2 \pi^2 D' + A},\tag{A10}$$

581 where  $s_0 = \int_0^1 S' dy'$  and  $s_n = 2 \int_0^1 S' \cos(n\pi y') dy'$  for  $n > 0$ .

582 A two-mode approximation suffices to illustrate key properties of the solution,

$$T(y') \approx \langle E \rangle + \frac{\alpha BF}{A} + G \cos \pi y' - \frac{2\alpha F}{\pi D'} \sin \frac{\pi y'}{2}, \quad (\text{A11})$$

583 where

$$G = \frac{2A \langle E \cos(\pi y') \rangle - 2\alpha BF/3}{\pi^2 D' + A} \quad (\text{A12})$$

584 and the angle brackets denote an integral over  $y'$ :  $\langle \cdot \rangle = \int_0^1 (\cdot) dy'$ .

585 The storm-track latitude is obtained by taking the second derivative of this solution with  
586 respect to  $y'$  and solving for its zeros. Using a first-order Taylor expansion in  $y'$ , the result  
587 is

$$y'_s \approx \frac{4GD'}{\alpha F}, \quad (\text{A13})$$

588 or, substituting for  $G$  and  $B$  from their definitions and restoring the original meridional  
589 coordinate,

$$y_s - y_h \approx \frac{8}{1 + \zeta} \langle E \cos(\pi y') \rangle \left( \frac{F}{D} \right)^{-1} - \frac{8}{3\pi^2} \frac{4 + \zeta}{1 + \zeta} (y_p - y_h), \quad (\text{A14})$$

590 where the eddy efficiency  $\zeta = \pi^2 \alpha^2 D/A$  measures the relative importance of eddy diffusion  
591 to radiative driving. For a fixed Hadley cell terminus latitude  $\phi_h$  (i.e., fixed  $y_h$  and  $\alpha$ ), it  
592 is clear from this solution that the storm tracks move closer to the Hadley cell terminus as  
593 the energy flux  $F$  across the terminus strengthens, or, more precisely, as the temperature  
594 gradient ( $\propto -F/D$ ) at the terminus strengthens at fixed  $\zeta$ . A second-order approximation,

$$y_s - y_h \approx \frac{1}{2} \left( \sqrt{\psi^2 + \frac{8}{\pi^2}} - \psi \right) (y_p - y_h), \quad (\text{A15})$$

595 where

$$\psi = \frac{3(1 + \zeta)}{4(4 + \zeta)} \left[ \frac{3\pi^2 \alpha \langle E \cos(\pi y') \rangle}{4 + \zeta} \left( \frac{F}{D} \right)^{-1} - 1 \right]^{-1}, \quad (\text{A16})$$

596 provides an improved fit (Fig. 3). Equation (A16) shows that  $\psi$  increases as the energy  
597 flux across the terminus strengthens. The first derivative of (A15) with respect to  $\psi$  is  
598 negative everywhere except in the limit as  $\psi$  goes to infinity, where the derivative goes to  
599 zero. Therefore, it is again clear that, for fixed  $y_h$  and  $\alpha$ , storm tracks move closer to the  
600 Hadley cell terminus when the strength of the energy flux across the terminus increases.

601

602

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TABLE 1. Simulation and reference parameters

Parameter	Symbol	Value
	Planet and fluid	
Specific Heat	$c_p$	$1004 \text{ J kg}^{-1} \text{ K}^{-1}$
Diffusivity	$D$	$2.1 \times 10^6 \text{ m}^2 \text{ s}^{-1}$
Hadley cell extent	$\phi_h$	$25^\circ\text{N}$
Planetary radius	$R$	$6.365 \times 10^6 \text{ m}$
Supercriticality	$S_{c,h}$	0.28
Pressure depth	$p_s - p_t$	700 hPa
Density	$\rho$	$1.0 \text{ kg m}^{-3}$
Convective lapse rate	$\gamma\Gamma_d$	$6.9 \text{ K km}^{-1}$
	Diabatic processes	
Radiative-equilibrium mean temperature	$\bar{T}_{eq}$	288 K
Pole-equator temp. contrast	$\Delta_H$	120 K
	Relaxation timescales	
Atmosphere	$\tau_{\text{rad}}$	50 days

TABLE 2. Simulation parameters for sensitivity tests using the analytical solution and for the numerical simulations [ min : (step) : max ]

Experimental Parameter	Symbol	Value
	Sensitivity study with analytical model	
Eddy efficiency	$\zeta' = D'/A = D\tau_{\text{rad}}(R[\pi/2 - \phi_h])^{-2}$	0.1 : 0.05 : 0.3
Subtropical northward eddy energy flux	$F$	0 : 30 K m/s
	Numerical simulations	
Convective parameter	$\gamma$	0.6 : 0.02 : 0.98

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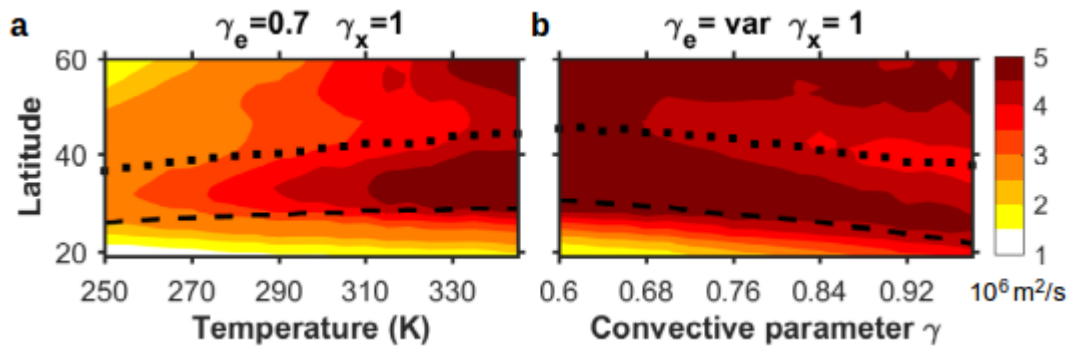


FIG. 1. Near-surface eddy diffusivity  $D = -\{\overline{v'\theta'}\}/\{\partial_y\theta\}$  for GCM simulations in which the mean radiative-equilibrium temperature and the tropical convective static stability are varied. (a) Variation of global-mean radiative-equilibrium temperature. Deep-tropical temperatures are relaxed toward a moist adiabatic profile ( $\gamma_e = 0.7$ ), while temperatures outside of this region are relaxed toward a dry adiabat ( $\gamma_x = 1$ ). (b) Variations of deep-tropical static stability ( $\gamma_e$ ), while the convective static stability in the extratropics remains dry adiabatic ( $\gamma_x = 1$ ). The dashed line marks the Hadley cell terminus, and the dotted line the storm track.

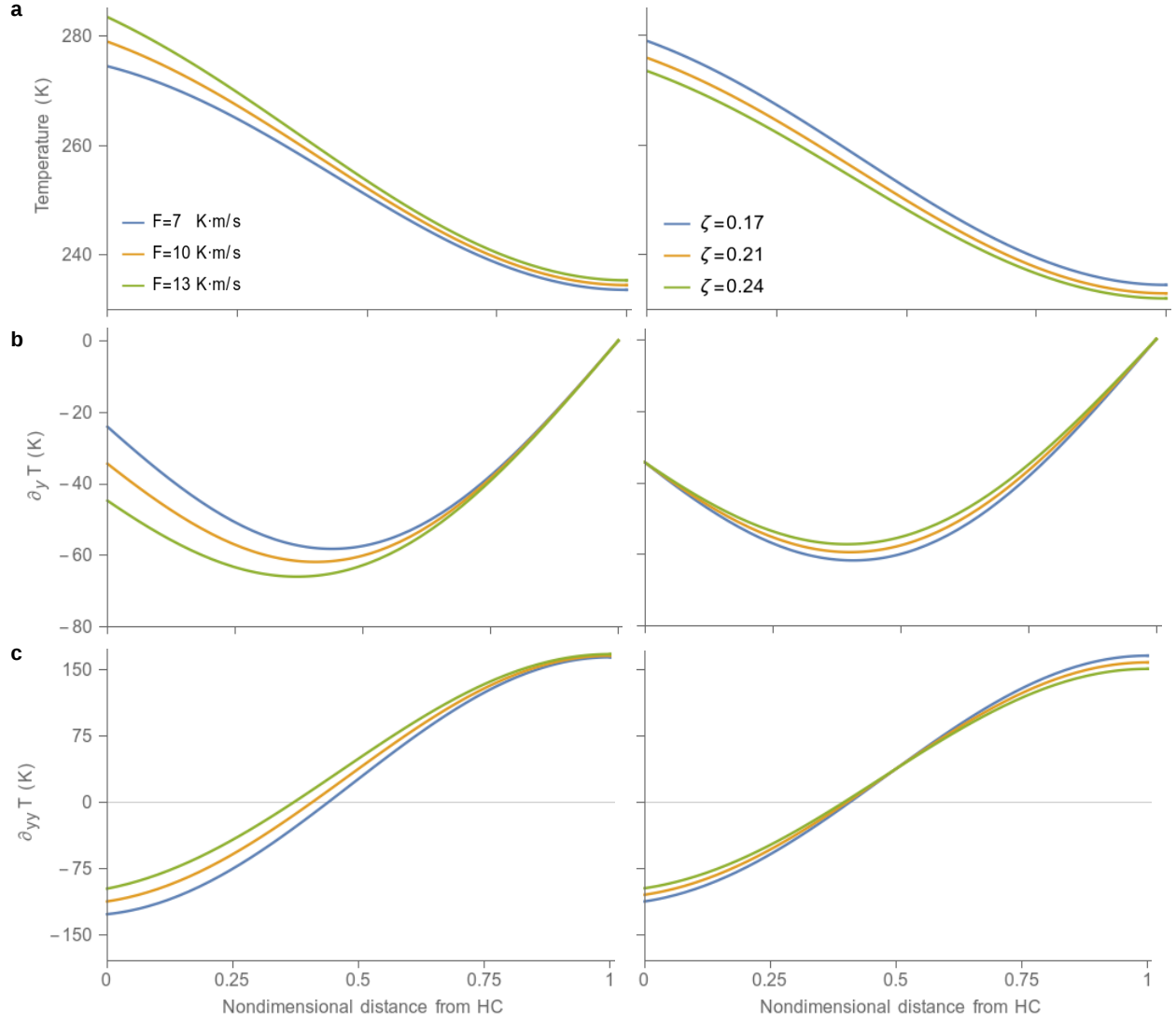


FIG. 2. Two-mode analytical approximations. Left: variations in subtropical energy fluxes for fixed eddy efficiency. Right: variations in eddy efficiency for fixed subtropical energy flux. (a) Zonal-mean temperature profile for a two-mode analytical approximation. (b) Zonal-mean meridional temperature-gradient profile. (c) Zonal-mean meridional temperature-curvature profile. Temperature gradients are multiplied by the Earth radius  $R$  and curvatures by  $R^2$  to express them in units of Kelvin. The Hadley cell terminus is at 0 and the pole at 1. Storm tracks are identified as global maxima of the absolute value of near-surface meridional temperature gradients, or as zeros of the temperature curvature. Strengthening the meridional temperature gradient at and thus the energy flux across the Hadley cell terminus shifts the storm tracks toward the Hadley cell terminus. For fixed energy flux across the Hadley cell, increasing the eddy efficiency shifts storm tracks toward the Hadley cell.

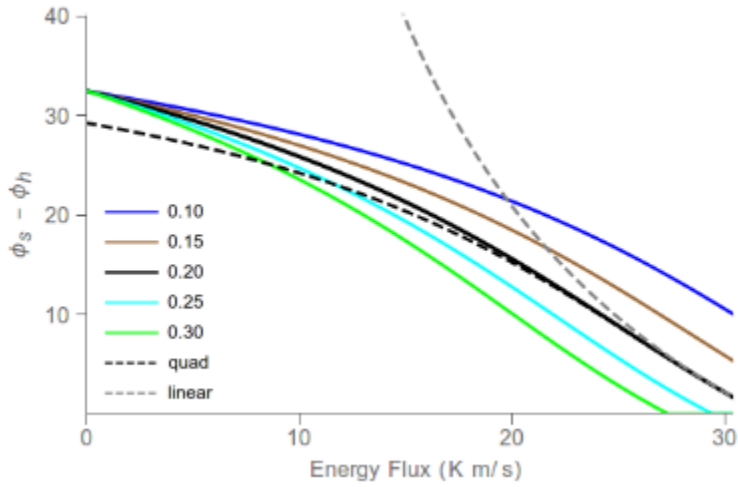


FIG. 3. Distance in degrees between the storm track latitude and the Hadley cell terminus latitude in the analytical solution to the EBM as a function of the meridional eddy energy flux  $F$  at the Hadley cell terminus, for 5 values of the eddy efficiency  $\zeta'$  between 0.1 and 0.3. The Hadley cell terminus is assumed constant at  $25^\circ\text{N}$ . The dashed gray line shows the first-order approximation (10) of the storm-track distance to the Hadley cell terminus for  $\zeta' = 0.2$ ; the dashed black line shows the second-order approximation (A16).

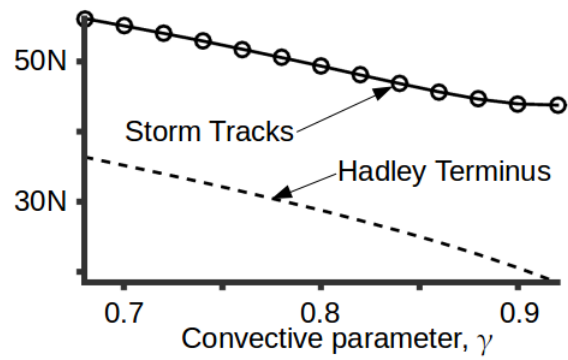


FIG. 4. Response of the full numerical solution of the EBM to changes in convective stability: Hadley cell terminus (dashed line) and storm tracks (solid line). As the convective stability decreases toward the right, the Hadley cell terminus shifts equatorward. For about  $\gamma \leq 0.88$ , the storm tracks shift poleward in tandem with the Hadley cell terminus; their shifts decouple for weaker convective stability ( $\gamma \geq 0.88$ ).

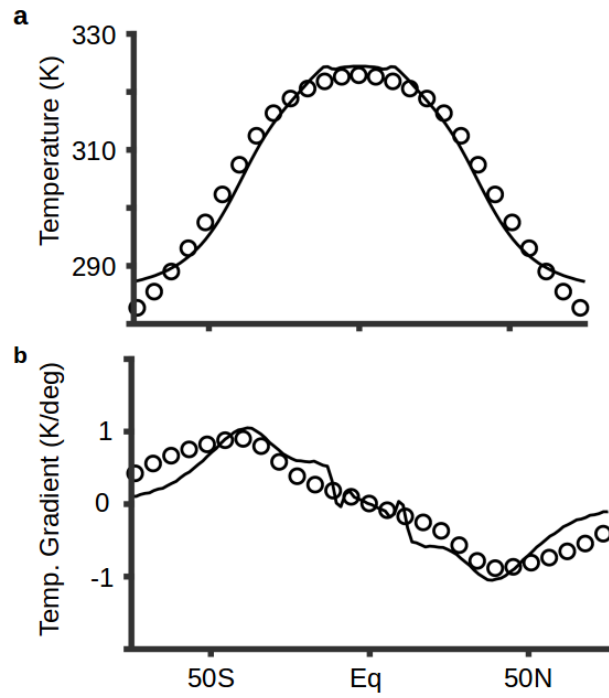


FIG. 5. Zonal-mean (a) temperature and (b) temperature gradient profiles from the GCM (solid lines) and the EBM (circles).

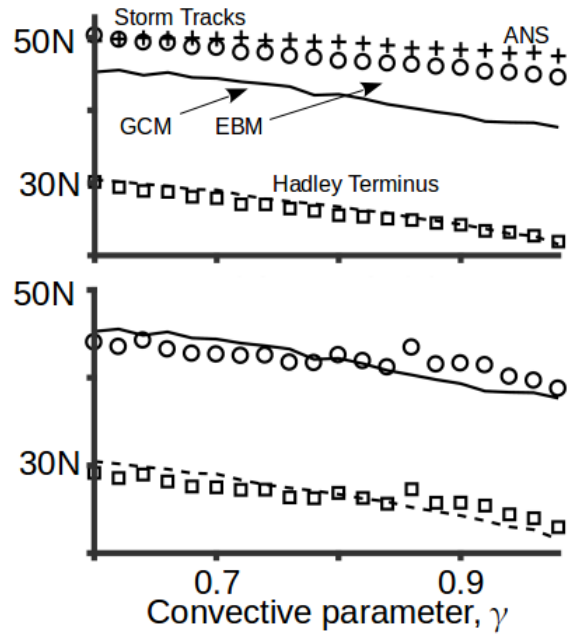


FIG. 6. (a) Storm track response to changes in deep-tropical convective stability in the idealized dry GCM (solid black line), in the full EBM with fixed tropical diffusivity (circles), and in the analytical solution of the EBM taking the Hadley cell terminus latitude in the full EBM as given (crosses). Note that deep-tropical stability is inversely proportional to the convective parameter,  $\gamma$ . The dashed line shows the Hadley cell terminus in the GCM and the squares show it for the EBM. (b) same as in (a), but with variable tropical diffusivity taken as the tropical average in each individual GCM simulation.



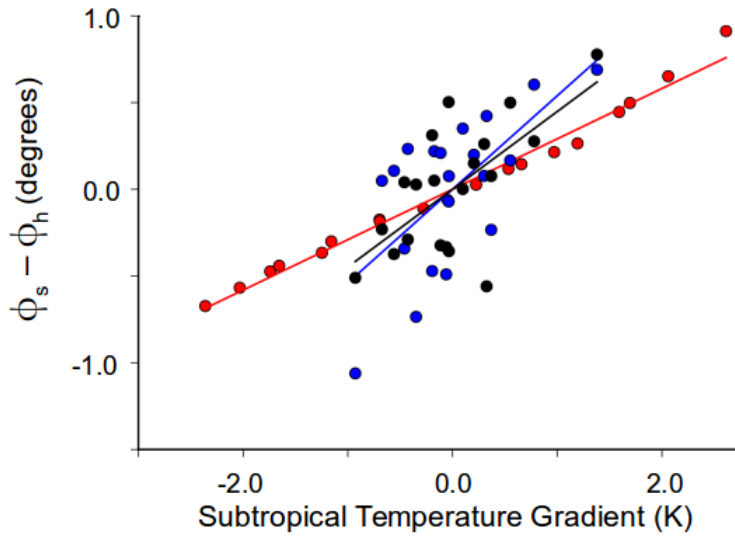


FIG. 7. Difference between the latitudes of the storm track  $\phi_s$  and the Hadley terminus  $\phi_h$  versus subtropical temperature gradient, for the EBM (red), the GCM (black), and the analytical EBM solution forced with parameters from the GCM but with the Hadley cell terminus assumed constant at  $25^\circ\text{N}$  (blue). The least-squares linear-regression lines are plotted in their respective colors. The temperature gradient has been multiplied by the Earth radius  $R$  to express it as a temperature difference in units of Kelvin. What are plotted are the anomalies relative to the arithmetic mean for each dataset.