Correction to "Single-layer axisymmetric model for a Hadley circulation with parameterized eddy momentum forcing"

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1. Introduction

[1] In the paper "Single-layer axisymmetric model for a Hadley circulation with parameterized eddy momentum forcing" by Adam H. Sobel and Tapio Schneider (*Journal of Advances in Modeling Earth Systems*, *1*, 10, doi:10.3894/JAMES.2009.1.10., 2009) [hereinafter referred to as SS09], we presented an axisymmetric single-layer model for Hadley circulations with parameterized eddy momentum fluxes. Results from this model were compared to results from the idealized general circulation model (GCM) of *Schneider and Bordoni* [2008]. Our axisymmetric model, as implemented in most of the paper, contained two errors. We correct these here.

2. Errors in Model Formulation and Implementation

[2] The model equations in SS09 should have been

$$\partial_t u - v\left(\beta y - \partial_y u\right) = -\mathcal{H}\left(\partial_y v\right)\left(\partial_y v\right)u - \mathcal{F} - \mathcal{S}, \qquad (1)$$

$$2\partial_t v + \beta y u = -\frac{gH}{T_0}\partial_y T, \qquad (2)$$

$$\partial_t \theta + \frac{\delta \Delta_z}{H}\partial_y v = \frac{\theta_E - \theta}{\tau}. \qquad (3)$$

[3] Here u and v are zonal and meridional velocities in an upper layer of thickness δ, and θ and T are the potential and actual temperature averaged throughout the depth of the troposphere; other notation is either standard, stated in SS09, or both. The meridional velocity v is assumed to be zero throughout the layer between $z=\delta$ and $z=H-\delta$, with H the tropopause height; in the bottom layer, $0 < z < \delta$, the meridional flow is assumed to be equal in magnitude and opposite in direction to that in the upper layer. The factor of 2 multiplying the first term on the left-hand side of (2) is correct only if the bottom and top layers have equal thicknesses; this assumption was not stated in SS09. Under this assumption, the meridional advection term $v\partial_y v$ should vanish; its presence in the model of SS09 was an error. Fortunately, this term was extremely small in all calculations in that paper, so that this error had no significant consequence. In the revised calculations shown here, this term has been removed from the model.

[4] The more significant error was not one of model formulation but of its implementation in the numerical code used for figures numbered 3 and higher in SS09. This was that θ was inadvertantly substituted for T on the right-hand side (RHS) of (2). This is the pressure gradient term. The two are related by $\theta = T(p_s/p_t)^{R/c_p}$, with the factor $(p_s/p_t)^{R/c_p} = 1.6$, so this error made the effective pressure gradient, and thus the computed circulations, somewhat too strong. Here we correct this error. The difference between the revised (correct) and original (incorrect) results is quantitatively significant, but it does not require revision of any of the main conclusions of the paper. However, the weaker circulations in the simple model are now in better agreement with the idealized GCM results.

3. Corrected Results

[5] Figures 1 and 2 of SS09 showed comparison of numerical calculations with the simple model to analytic solutions for a near-inviscid case. In these calculations, the second error described above—substitution of θ for T on the RHS of (2)—was not made. The code in these cases was thus correct apart from the inclusion of the meridional advection term in (2). The latter error has no visible effect on the plotted results. Thus, we do not present revised versions of Figures 1 and 2.

[6] Figures 1 and 2 here are revised versions of Figures 4 and 5 in SS09. The thermal forcing is given by equation (3.2) of SS09. The parameterized eddy momentum flux divergence F vanishes, vertical advection of zonal momentum (the first term on the RHS of equation (2.1)) is included, and different values of the background Rayleigh drag coefficient on the zonal wind ϵ_u are used. These figures resemble the original figures qualitatively, including their dependence on ϵ_u. However, the circulations are weaker than in SS09.

[7] Figure 3 presents results from a set of calculations in which y_0 is varied from 0 to 2000 km in intervals of 200 km, as in Figure 6 of SS09. The change here is

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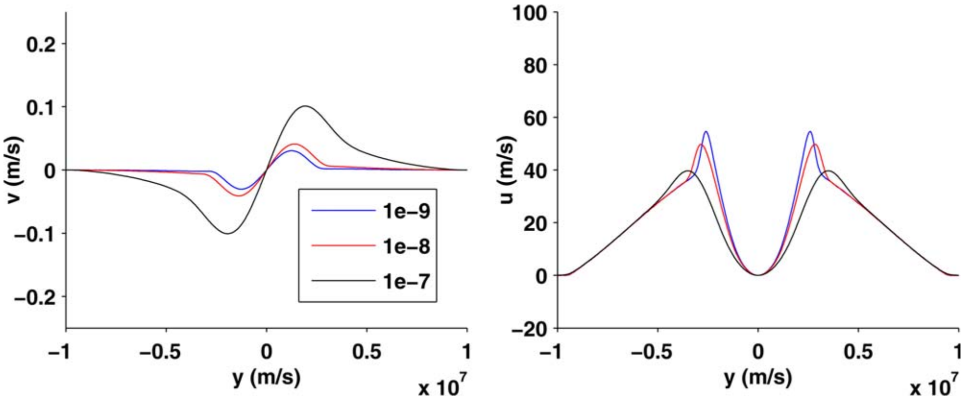

Figure 1. As in Figure 4 of SS09: Axisymmetric model circulation with thermal forcing as in equation (3.2) of that study with $y_0 = 0$, vertical advection of zonal momentum included, but no eddy momentum flux divergence.

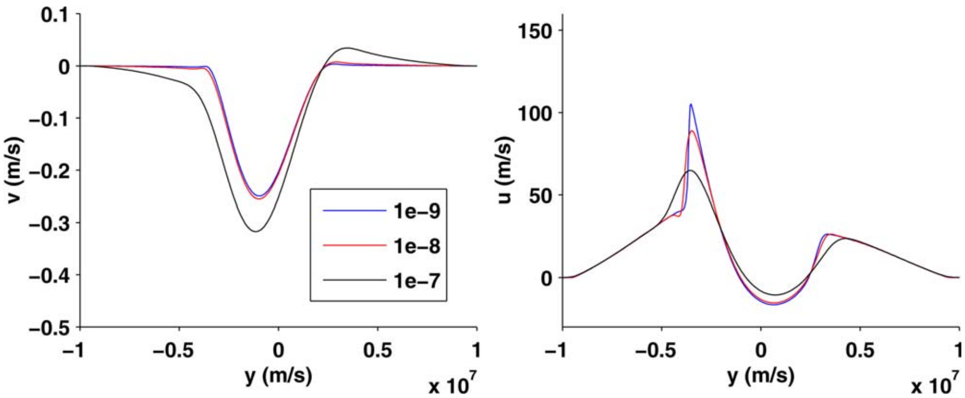

Figure 2. As in Figure 1, but with $y_0 = 1000\text{km}$.

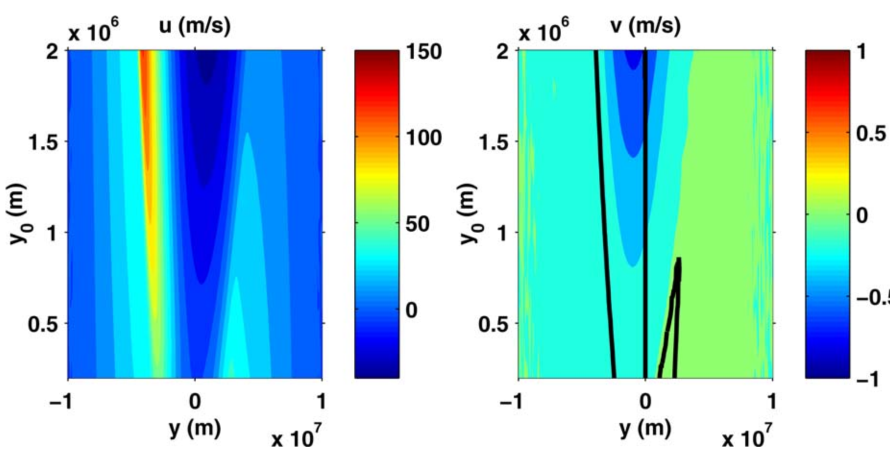

Figure 3. Plots of (left) zonal velocity, u (ms^{-1}), and (right) meridional velocity, v (ms^{-1}), as a function of latitudinal position, y, on the horizontal axis and of the θ_E maximum, y_0, on the vertical axis. In the plot of v, heavy black contours enclose regions of $R_o = -\zeta/(\beta y) > 0.6$.

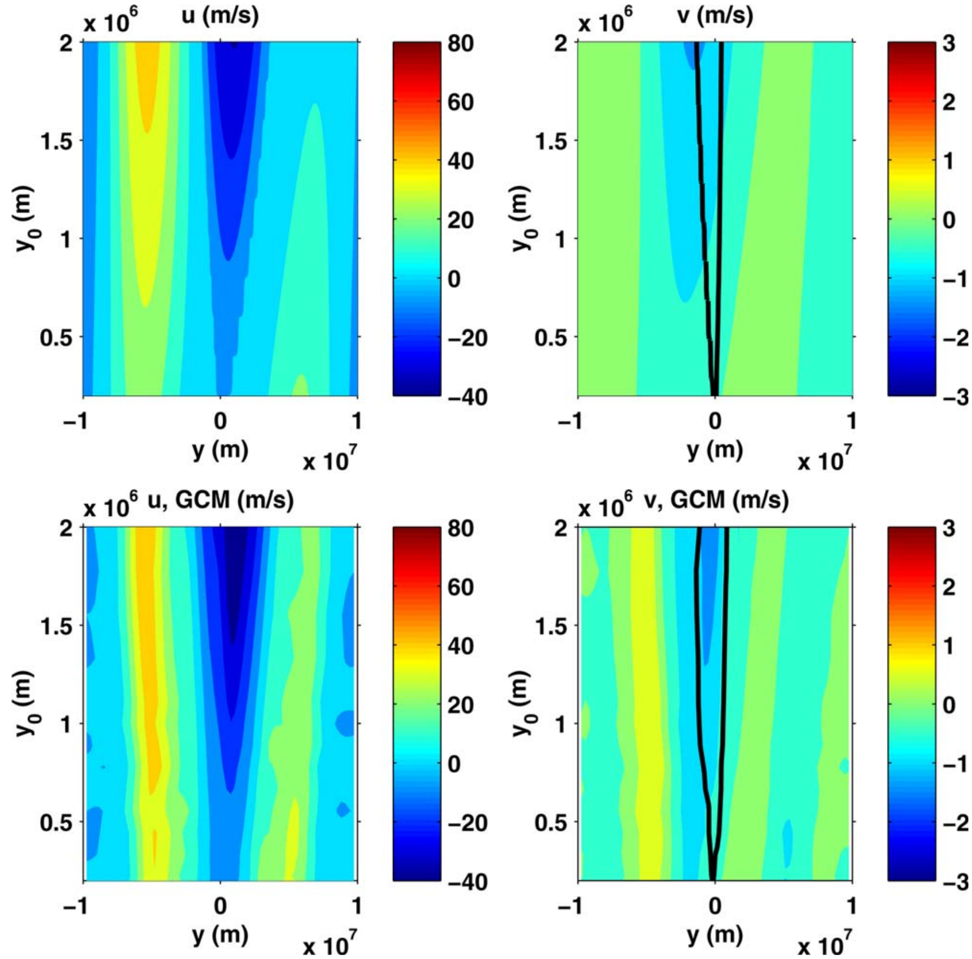

Figure 4. As in Figure 3, but for calculations with EMFD included. GCM results are shown on the bottom row.

quantitative but not qualitative. The circulation becomes weaker in both its meridional and zonal components. The maximum value of v remains within the region where $R_o > 0.6$ for all y_0.

[8] Figure 4 presents, in the top two plots, a set of calculations identical to that in Figure 3, except that $\mathscr{S}$ is nonzero and parameterized according to equation (2.5) in SS09, with $v_d = 2.5 \quad \mathrm{m\,s^{-1}}$, as in that paper. This is a

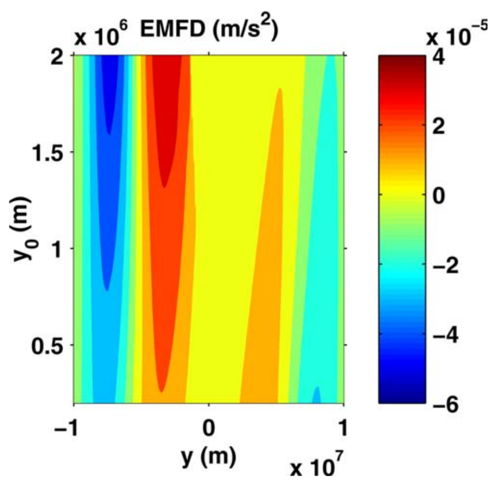

Figure 5. Eddy momentum flux divergence ($\mathrm{ms^{-2}}$) from the single-layer model. The white color refers to positive values larger than 4×10^{-5}.

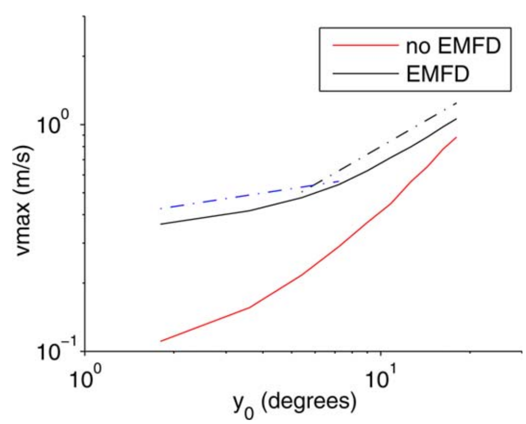

Figure 6. Log-log plot of maximum meridional velocity as a function of y_0, for the calculations with eddy momentum flux divergence (black) and without (red). Reference power laws of $y_0^{1/5}$ and $y_0^{3/4}$ are shown by the blue and black dot-dash lines respectively.

corrected version of Figure 6 in SS09. The lower two plots show results from the idealized GCM [*Schneider and Bordoni*, 2008], and are unchanged from SS09. As with the preceding figure, the circulation in the axisymmetric model is weaker but qualitatively unchanged after correction of the error.

[9] Figure 5 shows the eddy momentum flux divergence from the axisymmetric model, plotted as a function of y and y_0, as in Figure 8 of SS09. Consistent with the preceding figure, the pattern is the same as in Figure 8 of SS09 but with smaller amplitude.

[10] Figure 6 displays a log-log plot of the maximum absolute value of v as a function of y_0 from the axisymmetric model with and without eddy momentum flux divergence, as in Figure 9 of SS09. The reference power laws of $y_0^{1/5}$ and $y_0^{3/4}$, those found to be a good fit to the GCM results by *Schneider and Bordoni* [2008], are also shown on the figure, as in SS09; they are located in the same position on the present figure as in Figure 9 of SS09. The y axis in the figure is extended down to lower values than in SS09 to capture the smaller values in the case without eddy momentum flux divergence. We see again that the axisymmetric model's circulations are weaker than those in SS09 (the axisymmetric model results with eddy momentum flux divergence were above the reference curve in that paper, but are below it here); however, they follow the same power laws with the same degree of agreement as in SS09.

4. Summary

[11] We have corrected two errors in SS09. One, the inclusion of the advection term in the meridional momentum equation, had an entirely negligible impact on the results. The other, the incorrect use of θ in place of T in the computation of the meridional pressure gradient, led to circulations that were too strong. In the corrected model the circulations are weaker but otherwise qualitatively similar to those in SS09; in particular, their structures and dependence on the forcing latitude are essentially unchanged. The corrected axisymmetric model results agree somewhat better with the GCM results of *Schneider and Bordoni* [2008] than did the original results in SS09.

[12] **Acknowledgments.** We are grateful to Tim Cronin, who found the error in the pressure gradient term, (by independently coding the axisymmetric model and attempting to reproduce our results), and pointed it out to us; and to Xavier Levine, who found the error in the derivation of the meridional momentum equation resulting in the erroneous inclusion of the horizontal advection term.

References

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