ESE 134: Cloud and Boundary Layer Dynamics (HW 4, due May 14):

Moist convection and thermodynamics.

1. Consider the moist static energy

$$\mathrm{MSE} = h + \Phi \tag{1}$$

where

$$h = c_{pm}(T - T_0) + q_v L_{v,0} - q_i L_{f,0} + (1 - q_t) R_d T_0 \tag{2}$$

is the specific enthalpy of moist air and $\Phi = gz$ is the geopotential (specific potential energy in Earth's gravitational field). (Notation as in the notes on the webpage.)

   (a) Show that the expression (2) is in fact the specific enthalpy of moist air, and that $h = I(q, T) + R_m T$, where I is the specific internal energy of moist air.

   (b) Conservation of energy in an adiabatic and frictionless moving fluid can be stated as

$$\frac{\partial(\rho e)}{\partial t} + \nabla \cdot ((\rho e + p)\mathbf{u}) = 0. \tag{3}$$

   (In the Boussinesq approximation, $\rho = \rho_0$, but we do not need to make that approximation here.) Here,

$$e = \frac{1}{2}\|\mathbf{u}\|^2 + \Phi + I \tag{4}$$

   is the total energy. Show that the kinetic energy term in the total energy is small relative to the internal energy in low-Mach number flows. Hint: the speed of sound in moist air is

$$c_s = \left(\frac{c_{pm}}{c_{vm}} R_m T\right)^{1/2}.$$

   (c) Neglecting the small kinetic energy, show that the flux term in the energy conservation law (3) is the flux of MSE.

   (d) Explain why, when we consider the atmospheric energy budget in a statistically steady state (e.g., averaged over several years) and integrated vertically over atmospheric columns, we focus on the MSE flux.

2. Define the terms: (i) lifted condensation level, (ii) level of free convection, (iii) level of neutral buoyancy. Please use equations in your definitions.