ESE 134: Cloud and Boundary Layer Dynamics (HW 5, due May 21):

**Stratocumulus mixed-layer model.** A stratocumulus mixed-layer model can be written in terms of three unknowns: the height of the mixed-layer ($h$), and two slab (mixed-layer) average moist conserved variables: specific total water ($q$) and liquid water static energy ($s = c_p T + gz - L q_l$):

\[
\begin{align*}
\frac{ds}{dt} &= V(s_0 - s) + E(s^+ - s) - \frac{\Delta R}{\rho} \\
\frac{dq}{dt} &= V(q_0 - q) + E(q^+ - q) \\
\frac{dh}{dt} &= E - Dh
\end{align*}
\]

Here, $s_0$ and $q_0$ are the surface values, $s^+$ and $q^+$ are the values in the free troposphere, $V$ is a turbulent wind velocity close to the surface ($V = C_d U$, where $C_d$ is the drag coefficient and $U$ is the wind speed), $D$ is the divergence, $\Delta R$ is the radiative cooling, and $\rho$ is the density of air.

To close the model, an equation for the entrainment rate $E$ is necessary. Since the turbulence in the stratocumulus mixed-layer is mainly driven by cloud-top radiative cooling, we will use a simple and physically plausible formulation, which suggests that the entrainment forced by turbulent mixing is proportional to the rate of the driving of the flow (radiative cooling $\Delta R$) and inversely proportional to the stability of the interface at cloud top, with proportionality constant $\alpha \sim O(1)$, i.e.,

\[
E = \alpha \frac{\Delta R/\rho}{s^+ - s}
\]

1. Find the steady state analytic solutions of this system of equations for the particular case of $\alpha = 1$, and for a generic $\alpha$.

2. What are the specific solutions for typical parameter values: $V = 0.01$ m s$^{-1}$, $D = 5 \times 10^{-6}$ s$^{-1}$, $\Delta R = 50$ W m$^{-2}$, $\rho = 1$ kg m$^{-3}$, $s^+ - s = 10$ kJ kg$^{-1}$, $q^+ = 5$ g kg$^{-1}$, SST = 293 K, and $q_0 = q_s(SST) = 14$ g kg$^{-1}$?

3. Investigate the sensitivity of the solutions to changes in the parameters $D$, $V$, $\Delta R$ and $s^+$. *Suggestion: compute derivatives of the 3 unknowns ($s$, $q$ and $h$) with respect to parameters $D$, $V$, $\Delta R$ and $s^+$.*

4. Do these sensitivities make physical sense? Please explain qualitatively.