ESE 134: Cloud and Boundary Layer Dynamics (HW 6, due June 2):

Stratocumulus cloud cover.

1. **Cloud base**. The base of stratocumulus clouds topping a well-mixed boundary layer is approximately at the LCL. That is, it is at the height z_b at which the total specific humidity $q_t(z_b)$ is equal to the saturated specific humidity $q^*(z_b)$. Assume that the subcloud layer is well mixed, so that the total specific humidity q_t in it is constant.

   (a) Express the saturation specific humidity $q^*(z_b)$ as a linear function of z_b by Taylor expansion around the surface value $q^*(z = 0)$. Solve the resulting expression for the cloud base height z_b.

   (b) Next, write the expression obtained under (a) in terms of the relative humidity RH near the surface, using the fact that the total specific humidity can be written as $q_t \approx \mathrm{RH}\, q^*(0)$.

   (c) Then, manipulate the expression obtained under (b) further by expanding the derivative $\partial q^*/\partial z$ using the Clausius-Clapeyron relation and the chain rule of differentiation. Specifically, replace the derivative with respect to z by derivatives with respect to temperature T and pressure p, and use the hydrostatic relation for $\partial_z p$ and your knowledge of the temperature lapse rate in a well mixed subcloud layer for $\partial_z T$. Derive an expression for z_b.

   (d) Give an approximation for z_b in typical subtropical conditions (surface temperature 290 K) that depends only on constants (i.e., a constant height scale characteristic of subtropical conditions) and relative humidity. (You can look up the numerical value of the saturation vapor pressure or saturation specific humidity online, or use the expression in the thermodynamics class notes to evaluate it.) What is the cloud base height z_b, according to this expression, for $\mathrm{RH} = 0.85$?

2. **Cloud thickness**. The liquid water specific humidity $q_l(z)$ is given by the saturation excess $q_l(z) = \max(0, q_t - q^*(z))$. That is, it is zero below cloud base and nonzero between cloud base at height z_b and cloud top at height z_t.

   (a) Find a linearized expression for q_l as a function of height z by Taylor expansion of the saturation excess around cloud base. Sketch $q_l(z)$.

(b) Explain why the liquid water path

$$\mathrm{LWP} = \int_{z_b}^{z_t} \rho q_l \, dz$$

is an approximately quadratic function of the geometric cloud thickness.